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Anthropogenic Climate Change in an Integrated Energy Balance Model of Global and Urban Warming

Kimitoshi Sato

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Abstract This paper presents an integrated energy balance model of global and urban warming in the attributes/functionings framework à la Gorman–Lancaster–Sen and proposes a *Global Warming Function* and an *Urban Warming Function*. Also presented is a concept of *Heat Island Integral*, which measures the difference of anthropogenic heat stocks between two regions. The model involves residents, producers such as offices and manufacturers, and landscape gardeners who play a very important role in cooling down by roof-top gardening and tree-planting activities, etc. It is shown that *urban warming tax/subsidy scheme* is designed as a Groves mechanism by the implementation theory.

Keywords Anthropogenic heat stocks · Attributes/characteristics à la Gorman–Lancaster · Biological attributes · Global warming function · Heat island integral · Landscape gardener · Microclimate · Sen's functionings · Urban warming function · Urban warming tax/subsidy scheme

JEL Classification H2 · Q24 · R13 · R14

1 Introduction

An urban heat island is represented by isoterms on a map joining points of equal temperature and it is an analogy of contours of above and beneath the sea. By using the concept of *Heat Island Integral* that I introduce below, formal definitions of a heat island and a cool island are given in Sect. 2.3.[1]

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[1] See Gartland (2008) for a comprehensive treatment of the urban heat island.

K. Sato (✉)
College of Liberal Arts, International Christian University, 3-10-2 Osawa, Mitaka-shi, Tokyo 181-8585, Japan
e-mail: Kmtsato@aol.com

In the past 100 years, the temperature has risen 0.9 °C in Japan and 3.2 °C in Tokyo.[2] Whereas the temperature increase of Paris and New York are, respectively, about 0.9 °C and 1.6 °C for the same period. Tokyo's urban warming rate was remarkable; it is above the rising rate of the past ten thousand years. The Tokyo Metropolitan Research Institute for Environmental Protection estimated that the heat emitted from cars and air conditioners has resulted in a temperature increase of 0.4 °C and from the loss of greenery it has increased by 1.4 °C in the past century. The green area of Japan was halved from 1930 to 1990, thus, there has been an increase in radiant heat and a decrease in radiation cooling. There is therefore a pressing need to manage the urban thermal pollution due to urban warming in many cities of Japan. See Sato (2013) for biohazards and health risks suffered from urban thermal pollution.

In this paper, let me present a model to provide an optimal amount of heat as an urban public good. In order for this model to be operational, necessity compels us to devise a tax/subsidy scheme to determine the sum of emissions of heat released by metropolises into the urban atmosphere. The urban environment should now be perceived to be an intergenerational public good that we have to protect. Large cities such as Tokyo, Osaka, Sendai, Hiroshima, Fukuoka, Nagoya, and Sapporo consume a huge amount of energy by using cars and air conditioners, hence, urban warming has now become notable. Metropolitan residents are to be involved in the problem of urban warming which is now confirmed to be caused by emissions of enormous heat, i.e., all the households living in cities could be polluters as well as victims of the warming climate. Any local government is in charge of controlling the total amount of heat emitted by residents and producers to keep the urban climate not too warm in the near future.

This paper proposes to consider urban atmosphere as a complex of intangible attributes such as heat and gases, and the goods composed of tangible attributes that are produced by firms, and green areas consisting of flowers and trees as biological attributes supplied by landscape gardeners.

The paper proceeds as follows. Section 2 presents an integrated energy balance model of global and urban warming in the attributes/functionings framework à la Gorman–Lancaster–Sen and proposes an *Urban Warming Function*. Also presented is the concept of *Heat Island Integral*, which measures the difference of anthropogenic heat stocks between two regions. The model involves residents, producers such as offices and manufacturers, and landscape gardeners, who play a very important role to cool down by roof-top gardening and tree-planting activities, etc. It is shown that an *urban warming tax/subsidy scheme* is designed as a Groves mechanism by the implementation theory. Finally some remarks follow, including some technological innovations.

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[2] Urban heat islands exist in Tokyo's central business districts (CBD) such as Otemachi, Shinjuku, and Shibuya. The Imperial Palace, the Meiji Shrine, and the Shinjuku Gyoen National Garden are *Cool Islands*, and the temperature of these areas remains rather low compared to the hotter spots mentioned above. Japan has warmed by 0.6 °C in these three decades. See Howard (1833, 1837) for the case of England and Renou (1862, 1868) for the case of France.

2 Urban Warming and Heat Island

2.1 A Model of Urban Warming and Heat Island

This section introduces the attributes/characteristics model based on the Gorman–Lancasterian theory where goods are regarded as a composition of characteristics.[3] All trace gases such as greenhouse gases (GHGs) can be interpreted as *gaseous attributes* in our framework, since they infinitesimally compose the urban atmosphere as an urban public good. Characteristics of urban climate are in order: pollution substances, solar radiation, cloudiness, precipitation, temperature, absolute and relative humidity, and wind velocity. These attributes compose the urban planetary boundary layer. This paper focuses upon heat as an attribute, since it is the main cause of urban warming. Before rushing into our theoretical model, let me introduce some basic concepts of urban warming and heat islands.

Let us consider a city where we analyze urban warming and heat islands and the related problems occurring in urban and suburban areas. For the sake of simplicity, assume that our city is composed of two areas, i.e., the urban area and the suburban area, which are divided into many regions, $\beta \in \mathbf{B} = \{1, \ldots, B\}$: the set of regions. Assume that any size whatever can be chosen for a region.

Let there be N residents indexed by $i \in \mathbf{N} = \{1, \ldots, N\}$: the set of inhabitants who live in both urban and suburban areas. Each individual emits heat and trace gases when consuming goods or services indexed by j and its set is $\mathbf{J} = \{1, \ldots, J\}$. For the sake of simplicity, it is assumed that each producer j supplies only one good j which is composed of C characteristics indexed by $c \in \mathbf{C} = \{1, \ldots, C\}$: the set of tangible attributes. Denote as q_{jc} an amount of attribute c embodied in one unit of good j. There are producers: e.g., offices, hotels, and manufacturers.

Suppose that resident i chooses a landscape gardener named ℓ to have a part of his/her land planted with trees and flowers, and $\boldsymbol{\Lambda} = \{1, \ldots, \Lambda\}$ is the set of gardeners. Define $q_{\ell s}$ as a biomass of species s in one cube meter supplied by landscape gardener ℓ, and $\mathbf{S} = \{1, \ldots, S\}$ as the set of species of flora and fauna as *biological attributes*. Let x_{ij} be resident i's consumption of good j, and $V_{i\ell}$ be his/her demand for the plants supplied by landscape gardener ℓ, thus, $x_i = (x_{i1}, \ldots, x_{iJ}, V_{i\ell})$ is his/her consumption vector. There is also a local government whose task is to reduce heat emissions by making effective use of an *urban warming tax scheme* as defined below. No need to mention, every inhabitant, producer, and landscape gardener resides in some house or construction in region β, so that an index β will be omitted hereafter in almost all the cases, except for describing some variables related to region β.

Different from von Thünen (1826), or the usual urban economic theory, our city is hypothesized as follows:

H1. The city is formed in a heterogeneous plain, where the climate differs among its regions.

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[3] For the New Consumer Theory, see Gorman (1956/1980), Gorman and Myles (1987), and Lancaster (1966, 1971, 1991). See also Hagen (1975), Drèze and Hagen (1978), and Pendleton and Shonkwiler (2001).

H2. The city is not necessarily circular and its center is called the Central Business District.
H3. Its urban and suburban transportation systems are available in any direction whatsoever.
H4. Residents commute to work for an office in the CBD or in suburbs.
H5. Landscape gardeners plant in a part of the lands of residents and producers.
H6. Manufacturers produce goods in the suburban area.

The urban atmosphere is regarded as a complex of gaseous attributes including GHGs, which are to be mainly generated by production and consumption activities. The amount of gases such as N_2 and O_2 are stationary, so I can focus upon heat and trace gases as attributes in this paper. Let $\mathbf{G} = \{C+1, \ldots, C+G\}$ be the set of trace gases which compose the urban atmosphere.

Taking urban warming into consideration, let us extend and generalize the framework developed by Sato (2006, 2008). Inhabitants emit heat and gases, $q_{ih} \geq 0$ and $q_{ig} \geq 0$, $\forall g \in \mathbf{G}$, which are resident i's unit emissions of heat and gas. Hence, $q_{ih}V_{i\ell}$ and $q_{ig}V_{i\ell}$, $\forall g \in \mathbf{G}$, are individual i's emitted quantities of heat and gas in his/her consumption of $V_{i\ell}$ units of greening service ℓ. Resident i's consumption of gas and heat as attributes are, respectively, given by

$$z_{ig} = q_{ig}\left(\sum_{j\in\mathbf{J}} x_{ij} + V_{i\ell}\right), \quad \forall g \in \mathbf{G} \tag{1}$$

and

$$z_{ih} = q_{ih}\left(\sum_{j\in\mathbf{J}} x_{ij} + V_{i\ell}\right). \tag{2}$$

When producing one unit of good, each producer cannot choose but jointly emit trace gases as vexing by-products, $q_{jg} \geq 0$, which is producer j's unit emission of gas. Thus, $q_{jg}x_j$ is j's amount emitted of gas when it produces x_j units of good j. $q_{jg}x_j$ and $q_{jh}x_j$ producer j's amounts of gas and heat in his/her production of goods. One observes therefore

$$z_{jg} = q_{jg}x_j. \tag{3}$$

Similarly, when producing one unit of good, each producer must jointly emit heat as an annoying by-product, $q_{jh} \geq 0$, which is producer j's unit emission of heat. Thus, $q_{jh}x_j$ is producer j's amount emitted of heat when it produces x_j units of good j. One obtains therefore

$$z_{jh} = q_{jh}x_j. \tag{4}$$

Landscape gardeners also emit heat and gases, $q_{\ell h} \geq 0$ and $q_{\ell g} \geq 0$, $\forall g \in \mathbf{G}$, which are gardener ℓ's unit emissions of heat and any gas. Hence, $q_{\ell g}V_\ell$ and $q_{\ell h}V_\ell$ are gardener ℓ's emitted quantity of a gas and heat, respectively, in his/her provision of V_ℓ units of greening service ℓ. We have

$$z_{\ell g} = q_{\ell g}V_\ell \tag{5}$$

and

$$z_{\ell h} = q_{\ell h} V_\ell. \tag{6}$$

Let z_{i0} be resident i's available time that he/she possesses, by which he/she utilizes other characteristics. In other words, any characteristic cannot be utilized without using time. Note that dioxide (CO_2) can be reduced by Π_g which is an amount of CO_2 fixed in trees, i.e., the net carbon gain.[4] Amounts of each tangible attribute embodied in the goods and intangible attributes in the atmosphere, which are consumed by metropolitan agents are given for any $c \in \mathbf{C}$, for any $g \in \mathbf{G}$, and for heat:

$$z_{ic} = \sum_{j \in \mathbf{J}} q_{jc} x_{ij}, \tag{7}$$

$$z_g = \sum_{i \in \mathbf{N}} z_{ig} + \sum_{j \in \mathbf{J}} z_{jg} + \sum_{\ell \in \boldsymbol{\Lambda}} z_{\ell g} - \chi_g \Pi_g, \tag{8}$$

$$\chi_g = 1 \quad \text{if } g = \mathrm{CO}_2 \quad \text{and} \quad \chi_g = 0 \quad \text{if } g \neq \mathrm{CO}_2,$$

and

$$z_h = \sum_{i \in \mathbf{N}} z_{ih} + \sum_{j \in \mathbf{J}} z_{jh} + \sum_{\ell \in \boldsymbol{\Lambda}} z_{\ell h}. \tag{9}$$

In the above equations, z_{ic} means the consumption of tangible attributes which compose goods, while z_g and z_h represent the total amount of a trace gas and heat emitted by all residents, producers and gardeners. Note also that the values of z_g, $\forall g \in \mathbf{G}$, and z_h can be measured in tons or kilojoules. Heat and gases are generated both in the consumption and production of goods and services. Every inhabitant is made to consume not only his/her emissions but also the quantity emitted by the rest of the city. When he/she uses goods, he/she emits heat and gases. They are already released when the goods are made by producers and greening services are provided by landscape gardeners.

The above three equations may be interpreted as *characteristics availability functions* which convert commodities into attributes. In the framework of this paper, any good j can be recognized as $(x_j, q_{j1}, \dots, q_{jC+G})$ and any greening service ℓ can be represented as $(V_\ell, q_{\ell C+1}, \dots, q_{\ell C+G}, q_{\ell C+G+1}, \dots, q_{\ell C+G+S})$. The amount of any characteristic associated with each good and service can be regarded as a parameter that is objective and common to all consumers, i.e., it has the public-good property. Thus, the inhabitants as consumers must behave as "price and quality takers," since they can only change their consumption of z_{ic}, z_{ih}, and z_{ig}, via the choice of x_{ij} and $V_{i\ell}$, given the price and the quality of each good and service. Producers and gardeners can choose the composition of attributes embedded in their goods and their greening services.

The following notation is used in what follows.

T: temperature (°C)

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[4] See Hof et al. (1990) and Sato (2006) for the fixation of CO_2 in trees.

Al: planetary albedo (0.3) determining how much of the incoming energy is reflected by the atmosphere
Ω: solar constant (1372 $\mathrm{W\,m^{-2}}$)
ε: emissivity (assumed to be 1)
σ: Stefan–Boltzmann constant ($5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$) [the outgoing flux is $\varepsilon\sigma T^4$ by the Stefan–Boltzmann Law]
K (kelvin): 273.15 K = 0 °C
Υ_β: anthropogenic heat stocks at region β [J]
$\dot{T}^t_\beta$: temperature increase at region β at time t [°C]
c_p: specific heat capacity [1004.2 J/kg °C]
u, v: coordinates [m]
M: atmospheric density [$1.293\ \mathrm{kg\,m^{-3}}$]
A_β: airshed of region β [m^3]
Ab: a coefficient (0.3) determining how much of the energy which is not absorbed by the surface of the earth

2.2 Global Warming Function

It is observed that warming of cities is mainly due to heat emissions and partly due to global warming, and the GHGs' concentrations affect the latter. These effects therefore must be incorporated in the model of urban and global warming.

As was mentioned in Sect. 2.1, z_g, $\forall g \in \mathbf{G}$, is the total quantity of each GHG released all over the world. A part, $\alpha_g z_g$, $0 < \alpha_g < 1$, $\forall g \in \mathbf{G}$, of an aggregate emission of trace gas g, is observed to go to the atmosphere and the rest, $(1 - \alpha_g) z_g$, is perceived to be absorbed by the oceans and forests as carbon sinks, if g is carbon dioxide. Of this amount, about 43 % of the CO_2 emissions are observed to be absorbed. The mass of the gth GHG staying in the atmosphere is $\alpha_g z_g$, $\forall g \in \mathbf{G}$. The disintegration rate or an inverse of an atmospheric lifetime of each trace gas is denoted as μ_g, $0 < \mu_g < \alpha_g$, $\forall g \in \mathbf{G}$.

One problem of the GHGs is that they are not flows (emissions), but stocks (concentrations). Let $t \in [0, \infty)$ be the time argument. Denote t_0 as a base year. Let $\nu_g > 0$, $\forall g \in \mathbf{G}$, be a conversion parameter from mass (GtC/year) to concentration (ppm), then the latter at time t is represented by

$$\zeta^t_g = \int_{t_0}^{t} \nu_g \bigl(\alpha^\tau_g - \mu^\tau_g\bigr) z^\tau_g \, d\tau, \quad \forall g \in \mathbf{G}. \tag{10}$$

So one observes a vector of GHGs' concentrations as stocks:

$$Z^t = \bigl(\zeta^t_1, \ldots, \zeta^t_G\bigr). \tag{11}$$

In the sequel, let me follow Greiner (2004a, 2004b) for global warming in an endogenous growth model, originally due to Roedel (2001).

The most basic energy balance model is presented as follows:

$$T = \left\{ \frac{\Omega(1 - Al)}{4\varepsilon\sigma} \right\}^{1/4} = \left(\frac{1372 \times 0.7}{4 \times 5.67 \times 10^{-8}} \right)^{1/4} = 255\ \mathrm{K} = -18\ ^\circ\mathrm{C}. \tag{12}$$

This is the temperature without the greenhouse effect. Following Greiner (2004a, 2004b), the difference between the outgoing flux and the incoming radiative flux is given by

$$F = \frac{19.95}{109}. \tag{13}$$

The average surface temperature on earth with the greenhouse effect can be calculated by

$$\begin{aligned} T &= \left\{ \frac{\Omega(1-Al)Ab}{4\varepsilon\sigma F} \right\}^{1/4} = \left(\frac{1372 \times 0.21}{4 \times 5.67 \times 10^{-8} \times (19.95/109)} \right)^{1/4} \\ &= 288\ \mathrm{K} = 15\ ^{\circ}\mathrm{C}. \end{aligned} \tag{14}$$

Thus, the above calculations lead us to conclude that the greenhouse effect for the earth is 33 °C. Next we incorporate the effect of GHGs' concentrations to global warming. Let ζ_{CO_2} be an actual concentration of CO_2 at time t and $\zeta^0_{\mathrm{CO}_2}$ its concentration at the Pre-industrial Revolution: e.g., the former is 377 ppm in 2004 and the latter is 280 ppm in 1750. IPCC (1990) assumed that the radiative forcing ($\mathrm{W\,m^{-2}}$) of carbon dioxide is given by

$$\varXi^t\bigl(\zeta^t_{\mathrm{CO}_2}\bigr) = 5.35 \ln\bigl(\zeta^t_{\mathrm{CO}_2}/\zeta^{1750}_{\mathrm{CO}_2}\bigr). \tag{15}$$

For the sake of simplicity, we take two main GHGs: CO_2 and nitrous oxide. Let ρ_g be a Global Warming Potential and μ_g is an inverse of an atmospheric lifetime compared with CO_2 ($\rho_{\mathrm{CO}_2} = 1$ and $\mu_{\mathrm{CO}_2} = 1$). According to Michaelis (1990), nitrous oxide's contribution to global warming is $\rho_g \mu_g = 58/12 = 29/6$ compared with CO_2. We remark that the global warming depends on the stocks (concentrations) Z^t of GHGs:

$$\varXi^t\bigl(Z^t\bigr) = 5.35 \sum_{g \in \mathbf{G}} \rho_g \mu_g \ln\bigl(\zeta^t_g/\zeta^{1750}_g\bigr). \tag{16}$$

It is observed that the average surface temperature on earth with global warming can be calculated by

$$\begin{aligned} T^{2004} &= \left[\frac{\Omega(1-Al)Ab + 5.35\{\ln(\zeta^{2004}_{\mathrm{CO}_2}/\zeta^{1750}_{\mathrm{CO}_2}) + (29/6)\ln(\zeta^{2004}_{\mathrm{N_2O}}/\zeta^{1750}_{\mathrm{N_2O}})\}}{4\varepsilon\sigma F} \right]^{1/4} \\ &= \left\{ \frac{1372 \times 0.21 + 5.35\{\ln(1.35) + (29/6)\ln(1.18)\}}{4 \times 5.67 \times 10^{-8} \times (19.95/109)} \right\}^{1/4} \\ &= 290\ \mathrm{K} = 17\ ^{\circ}\mathrm{C}. \end{aligned} \tag{17}$$

Compare the two values of temperature, i.e., 15 °C and 17 °C. The difference is 2 °C, which is attributable to the effect of GHGs' concentrations between the years 1750 and 2004.

The above arguments lead us to define the *Global Warming Function,*

$$W\bigl(\varXi^t\bigl(Z^t\bigr)\bigr) = \left\{ \frac{\Omega(1-Al)Ab + \varXi^t(Z^t)}{4\varepsilon\sigma F} \right\}^{1/4}, \tag{18}$$

which represents the average surface temperature on earth with the global warming due to the buildup of GHGs' concentrations.[5]

2.3 Heat Island Integral

The problem of how to represent anthropogenic heat stocks in any block was analyzed in Sato (2006) who introduced the concept of *Heat Island Integral*. Here a new version is proposed. In effect, there are differences in the temperature of building surface, back alleys, roof-tops, streets, and green tracts of land, which are directly exposed to the solar radiation. These differences of the surface temperature of the ground coverage can be measured by utilizing infrared cameras or remote sensing.

A formula is proposed for heat as a stock in this subsection. It is the *microclimate* surrounding a construction, which most influences any agent who resides or works in any region. However, climatical incidents depend not only upon the heat stocks in each region, but also upon those in the entire city. More precisely, it is the sum of developable areas of the ground coverage, e.g., streets, tree crowns, roof-tops, and walls of the buildings which exist in region β. Denote κ_β^r and $\delta_\beta^r(u, v)$ as the area and the height of construction r in region β, respectively, and $\mathbf{R}$ as the set of constructions. Suppose β is an urban commercial region and β' is a suburban residential region.

The Riemann sum has led me to propose a concept of *Heat Island Integral* (*HII*) between regions β and β':

$$\begin{aligned} HII = c_p M \sum_{r \in \mathbf{R}} \bigg\{ & A_\beta \dot{T}_\beta \iint_{\kappa_\beta^r} \delta_\beta^r(u, v)\, du\, dv \\ & - A_{\beta'} \dot{T}_{\beta'} \iint_{\kappa_{\beta'}^r} \delta_{\beta'}^r(u, v)\, du\, dv \bigg\}. \end{aligned} \tag{19}$$

Needless to say, the existence condition of this multiple integral is that the functions are continuous and compact in the domains κ_β^r and $\kappa_{\beta'}^r$, and it is easily seen that this condition is satisfied. Region β is called a *Heat Island* if $HII > 0$. Naturally, it is necessary to consider meteorological conditions peculiar to region β, such as the Foehn phenomenon, the convergence of sea breezes, and the transport of the warmed air from other regions, as well as the configuration of the region such as being a basin. Region β is called a *Cool Island* if $HII \leq 0$.

As $\dot{T}_\beta = \Upsilon_\beta / c_p A_\beta M$ by physics, *HII* can be rewritten as

$$HII = \sum_{r \in \mathbf{R}} \bigg\{ \Upsilon_\beta \iint_{\kappa_\beta^r} \delta_\beta^r(u, v)\, du\, dv - \Upsilon_{\beta'} \iint_{\kappa_{\beta'}^r} \delta_{\beta'}^r(u, v)\, du\, dv \bigg\}. \tag{20}$$

Remark 1 The difference between Heat Island Intensity and Heat Island Integral is obvious. The former shows the difference of the temperature between two regions,

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[5] See Schneider (1989) and Graedel and Crutzen (1995) for comprehensive knowledge about the atmosphere and climate change. Sato (2007) presented a model of global warming in the Gorman–Lancaster–Sen framework.

while the latter signifies that of the anthropogenic heat stocks between them. The temperature of asphalt and concrete is very often higher than that of the atmosphere in the summertime. It is observed that these materials absorb approximately 70 % of the solar radiation, so that the walls of buildings made of concrete absorb the same amount of heat, which results in urban warming. Thus, an area with a lot of these constructions is apt to become warmer. Notice that concrete is a material which is not at all "soon hot, soon cold," but hard to be heated and hard to cool off.

2.4 Urban Warming Function

Let $z_{ih\beta}$ be heat emitted by resident i and $z_{jh\beta}$ ($z_{\ell h\beta}$, resp.) is heat emissions of producer j (landscape gardener ℓ, resp.) at region β. Any city is composed of regions with anthropogenic heat stocks, $\varUpsilon_\beta = \varUpsilon_\beta(E_\beta)$, where $E_\beta = \sum_{i\in\mathbf{N}} z_{ih\beta} + \sum_{j\in\mathbf{J}} z_{jh\beta} + \sum_{\ell\in\Lambda} z_{\ell h\beta}$, $\forall\beta \in \mathbf{B}$. With $E = (E_1, \dots, E_B)$, the total heat stocks, $\varUpsilon(E) = \sum_{\beta\in\mathbf{B}} \varUpsilon_\beta(E_\beta)$, may affect agents in any city.

As above, an increase in temperature of an airshed of region β is given by the following equation: $\forall\beta \in \mathbf{B}$

$$\dot{T}_\beta^t = \frac{\text{anthropogenic heat stocks}}{\text{specific heat capacity} \times \text{airshed of region } \beta \times \text{atmospheric density}}. \tag{21}$$

In August 2004, the 23 wards of Tokyo generated an anthropogenic heat stocks of 2106.5 TJ/day. It was observed that warm air occupied up to an altitude of 400 m in the summer of Tokyo of which area is 621 km^2. We choose this as an airshed and the altitude of 400 m, then the average daily temperature increase is given by

$$\dot{T}_{\text{Tokyo23}}^{\text{Aug.2004}} = \frac{2106.5\ \text{TJ}}{1004.2\ \text{J/kg}\,{}^\circ\text{C} \times 621 \times 10^6\ \text{m}^2 \times 400\ \text{m} \times 1.293\ \text{kg}\,\text{m}^{-3}}. \tag{22}$$

Adding the above equation to the model to compute the average surface temperature on earth, and one observes

$$\begin{aligned} &U_{\text{Tokyo23}}^{\text{Aug.2004}} \\ &\quad = \left[\frac{\varOmega(1-Al)Ab + 5.35\{\ln(\zeta_{\text{CO}_2}^{2004}/\zeta_{\text{CO}_2}^{1750}) + (29/6)\ln(\zeta_{\text{N}_2\text{O}}^{2004}/\zeta_{\text{N}_2\text{O}}^{1750})\}}{4\varepsilon\sigma F}\right]^{1/4} \\ &\qquad + \frac{\varUpsilon(E^{2004})}{c_p \times A_{\text{Tokyo23}} \times M} \\ &\quad = 17\ {}^\circ\text{C} + \frac{2106.5 \times 10^{12}\ {}^\circ\text{C}}{1004.2 \times 621 \times 10^6 \times 400 \times 1.293} \\ &\quad = 20.26\ {}^\circ\text{C}. \end{aligned} \tag{23}$$

The difference between this temperature and the average temperature is 3.26 °C. It is considered to be attributable urban warming of the average daily anthropogenic heat stocks generated from the 23 wards of Tokyo in August 2004.

The greenhouse effect, global warming, and urban warming of any region $\beta \in \mathbf{B}$ is involved in the function:

$$U_\beta^t = \left\{ \frac{\varOmega(1 - Al)Ab + \varXi^t(Z^t)}{4\varepsilon\sigma F} \right\}^{1/4} + \frac{\varUpsilon_\beta^t(E_\beta^t)}{c_p A_\beta M}, \tag{24}$$

which represents the average surface temperature of any region β. In the bracket the denominator is given, and the second term varies according to each region. With the influx and outflux of energy to earth, the concentrations of GHGs and anthropogenic heat stocks define the *Urban Warming Function* U_β^t. Note that $\varXi^t$ and $\varUpsilon_\beta^t$ are functions of stocks of GHGs and anthropogenic heat, respectively. The following assumption is needed.

Assumption 1 For any $\beta \in \mathbf{B}$, U_β^t is concave and twice continuously differentiable.

Remark 2 It can easily be seen that $\partial U_\beta^t/\partial \varXi^t > 0$, $\partial^2 U_\beta^t/(\partial \varXi^t)^2 > 0$, $\partial \varXi/\partial \zeta_g > 0$, $\partial^2 \varXi^t/(\partial \zeta_g^t)^2 > 0$, $\forall g \in \mathbf{G}$, $\partial U_\beta^t/\partial \varUpsilon_\beta^t > 0$, and $\partial^2 U_\beta^t/(\partial \varUpsilon_\beta^t)^2 > 0$, $\forall \beta \in \mathbf{B}$.

Urban warming is a typical example of a public good which is non-rival but excludable. Its impact on each resident varies from region to region, which can be treated as a *regional public good*. The global warming function depends upon the anthropogenic heat stocks in region β, $\varUpsilon_\beta$, and the concentrations Z composed of gaseous attributes of the urban atmosphere.

2.5 Beings and Functionings of Residents

The Gorman–Lancasterian characteristics theory is the most suitable to analyze goods and the urban atmosphere which is perfectly divisible and decomposable as various attributes. However, the characteristics availability functions can be applied to any resident whose utilization differs from person to person. Consequently, each inhabitant's *functionings* should be introduced as one of the important concepts à la Sen (1985) to fully appraise the value of goods or their characteristics. Each resident's physical and climatical situations differ, so I must introduce the *functionings* which are represented below.

With time z_{i0}, the amounts of each characteristic embodied in the goods, the plants in the green area, the atmosphere, and the greenery consumed by resident i is given by

$$z_i = \bigl(z_{i0}, z_i^C, z_i^G, z_i^S, z_{ih}\bigr), \tag{25}$$

where $z_i^C = (z_{i1}, \dots, z_{iC})$, $z_i^G = (z_{iC+1}, \dots, z_{iC+G})$ and $z_i^S = (z_{iC+G+1}, \dots, z_{iC+G+S})$.

Denote $z = (z_1, \dots, z_N)$ and $U = (U_1, \dots, U_B)$. With consumption externality, beings of inhabitant i may be representable as

$$b_i = b_i\bigl(f_{i1}(z, U), \dots, f_{iK_i}(z, U)\bigr). \tag{26}$$

Let $\mathbf{K}_i = \{f_{i1}, \dots, f_{ik}, \dots, f_{iK_i}\}$, where K_i differs among individuals. The following assumption is needed.

Assumption 2 For any $i \in \mathbf{N}$, $f_{ik} \in \mathbf{K}_i$ is twice continuously differentiable.

2.6 Resident's Happiness Function and Valuing Well-Beings

Let any inhabitant i have his/her *Happiness Function* which is assumed to depend upon his/her being, thus, one observes

$$H_i = H_i(b_i). \tag{27}$$

Any person's use of functionings can vary his/her being and happiness. Let $\partial f_{ik}/\partial U_\beta$ and $\partial U_\beta/\partial \Xi$ be f_{ikU_β} and $U_{\beta\Xi}$, respectively. Let also $\partial \Xi/\partial \zeta_g$ and $\partial U_\beta/\partial \Upsilon_\beta$ be Ξ_g and $U_{\beta\Upsilon}$, respectively. Denote $\partial \zeta_g/\partial z_{ig}$ and $\partial \Upsilon_\beta/\partial z_{ih}$ as ζ_{gz} and $\Upsilon_{\beta z}$, respectively.

Define resident i's *hedonic shadow price* or *hedonic marginal willingness-to-pay* (HMW) of any tangible and intangible characteristic c and of any biological attribute s in greenery, $\forall c \in \mathbf{C}$, $\forall g \in \mathbf{G}$, $\forall s \in \mathbf{S}$:

$$\pi_{ic} = \frac{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{ic})}{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0})}, \tag{28}$$

$$\pi_{ig} = \frac{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{ig} + \sum_{\beta\in\mathbf{B}} f_{ikU_\beta} U_{\beta\Xi}\Xi_g\zeta_{gz})}{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0})}, \tag{29}$$

$$\pi_{is} = \frac{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{is})}{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0})}, \tag{30}$$

and

$$\pi_{ih} = \frac{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{ih} + \sum_{\beta\in\mathbf{B}} f_{ikU_\beta} U_{\beta\Upsilon}\Upsilon_{\beta z})}{\sum_{k\in\mathbf{K}_i}(dH_i/db_i)(\partial b_i/\partial f_{ik})(\partial f_{ik}/\partial z_{i0})}. \tag{31}$$

Remark 3

(i) π_{ih} is a marginal contribution of heat as an attribute to resident i's *marginal happiness* through his/her functionings in terms of his/her available time z_{i0}. It may correspond to a marginal rate of substitution between each attribute and the time z_{i0} in the utility theoretical context.

(ii) Whether inhabitant i considers a commodity or a service as good, irrelevant, or bad to his or her functionings is confirmed by the sign of $\sum_{c\in\mathbf{C}\cup\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc}$ for each good j and $\sum_{c\in\mathbf{S}\cup\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc}$ for any service ℓ. Moreover, whether a commodity or a service is socially good or not may also be examined by summing over individuals of $\sum_{c\in\mathbf{C}\cup\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc}$ for any good j and $\sum_{c\in\mathbf{S}\cup\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc}$ for any service ℓ. Now, to keep the room open to the cool air, an air conditioner is one of the necessities for our comfortable urban life under urban warming, even if it does emit heat as a vexing by-product. Consequently, $\pi_{ih} < 0$ holds for many people who feel displeased by the scorching heat due to urban warming. However, they wish to avoid heat, thus, $\sum_{c\in\mathbf{C}\cup\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc} \gg 0$ still prevails for j if it is an air conditioner. It cannot be helped, since everybody wants to keep his/her room cool, especially under a burning sun. It is not only heat but also the resulting high humidity, and actual and sensory high temperatures, that could lower some functionings of residents. This fact can be represented by $(dH_i/db_i)(\partial b_i/\partial f_{ik})\sum_{\beta\in\mathbf{B}} f_{ikU_\beta}U_{\beta\Upsilon}\Upsilon_{\beta z} < 0, \forall f_{ik}\in\mathbf{K}_i, \forall i\in\mathbf{N}$.

3 Optimizations Under Urban Warming and Heat Island

3.1 Residents

Consider that city dwellers know the risks of human-induced future urban climate changes, and that they have an incentive to optimize the composition of the urban atmosphere in order to aim at achieving their personal best-beings in the city where they live and work.

Let e be a generic index for any emitter and $\mathbf{E}$ the set of all emitters in any city. Since heat and gases are generated by the use of fossil fuels, let ε_e be emitter e's integrated emissions of heat and gases and denote $\varepsilon_{-e} = (\varepsilon_1, \ldots, \varepsilon_{e-1}, \varepsilon_{e+1}, \ldots, \varepsilon_{N+J+\Lambda})$. Urban warming is a "privately provided regional public good" that is supplied by individual emitters of heat in the atmosphere. By emitting heat, which means using the urban atmosphere, every metropolitan infinitesimally contributes to hot air as an urban public good. The inhabitants and producers in an urban heat island are both members who are forced to join in an "urban club."

Denote $\nu_i(U(\varepsilon_i(\alpha_i), \varepsilon_{-i}))$ as individual i's damage function. A variable $\alpha_i \geq 0$ is the abatement cost associated with his/her effort to avoid damages due to too much warmed urban air by reducing heat and gas emissions. It is assumed that α_i is known only to individual i. Functionings to cool cities are, for example, economizing in power by reducing the use of electrical fittings and by using a bicycle or a public transportation system instead of a private car. As urban warming accelerates, city dwellers will want to buy newly developed, more efficient air conditioners, which result in reduction of heat emissions. These are examples of functionings which explain α_i. Resident i has to pay $t_i(\varepsilon_i(\alpha_i))$ as an *urban warming tax*, in order for the urban atmosphere not to be warmed so much as to be unendurable to live in region β. It is natural to consider $t_i(\varepsilon_i(\alpha_i)) < t_i(\varepsilon_i(0))$, since paying an abatement cost could reduce emissions and his/her tax, which can incite people to behave environment-friendly.

Greening lands and roof-tops belong to the most effective ways to cool microclimates. The metropolitan government determine $0 < \varphi < 1$ as the refund rate for planting the vegetation $V_{i\ell}$ that resident i requests landscape gardener ℓ. Assume that the value of φ is decided by the scientific data about tree-planting. As defined, $\sigma_{i\ell}$ is landscape gardener ℓ's greening cost per cubic meter of inhabitant i's garden, hence, $\sigma_{i\ell}V_{i\ell}$ is resident i's greening cost, and $\varphi\sigma_{i\ell}V_{i\ell}$ is i's refund for the effort to plant trees in his/her garden.

The set $\mathbf{J}$ includes all the goods and services such as electricity and water. Let p_j be a unit price of good j, then each inhabitant's budget constraint is given by

$$z_{i0} = \sum_{j\in\mathbf{J}} p_j x_{ij} + \sigma_{i\ell}V_{i\ell} - \varphi\sigma_\ell V_{i\ell} + t_i\big(\varepsilon_i(\alpha_i)\big) + \nu_i\big(U\big(\varepsilon_i(\alpha_i), \varepsilon_{-i}\big)\big). \tag{32}$$

The left-hand side of the above equation signifies the value of available time whose price is normalized to unity. An assumption is needed.

Assumption 3 For any $i \in \mathbf{N}$, t_i, ν_i, and ε_i are convex, and they are continuously differentiable, with $\partial \nu_i / \partial U_\beta > 0$, $\partial U_\beta / \partial \varepsilon_i > 0$, $d\varepsilon_i / d\alpha_i < 0$, and $dt_i / d\varepsilon_i > 0$, $\forall \beta \in \mathbf{B}$.

The more heat is emitted, the more the urban air warms, and the more costly life becomes. This signifies that the more damages there are due to urban warming augment, the more residents have to pay for cool air, the more tax they have to pay for alleviating the urban warming. Each resident's effort can diminish his/her tax payment. The maximand is the personal happiness function, thus, each resident solves the following optimization problem:

$$\begin{aligned}
&\text{Max } H_i = H_i(b_i)\\
&\quad \text{s.t.} \quad b_i = b_i\big(f_{i1}(z, U), \ldots, f_{iK_i}(z, U)\big)\\
&\qquad\quad U_\beta = \left\{\frac{\Omega(1 - Al)Ab + \Xi(Z)}{4\varepsilon\sigma F}\right\}^{1/4} + \frac{\Upsilon_\beta(E_\beta)}{c_p A s_\beta M}, \quad \forall \beta \in \mathbf{B},\\
&\qquad\quad z_{i0} = \sum_{j\in\mathbf{J}} p_j x_{ij} + \sigma_{i\ell} V_{i\ell} - \varphi\sigma_{i\ell} V_{i\ell} + t_i\big(\varepsilon_i(\alpha_i)\big) + \nu_i\big(U\big(\varepsilon_i(\alpha_i), \varepsilon_{-i}\big)\big),\\
&\qquad\quad x_{ij} \geq 0, \quad \forall j \in \mathbf{J},\\
&\qquad\quad V_{i\ell} \geq 0, \quad \forall \ell \in \boldsymbol{\Lambda}.
\end{aligned}$$

The FOCs of the above optimization problem are in order. For any resident $i \in N$, an individually optimal consumption of goods composed by Gorman–Lancasterian tangible and accompanying intangible attributes, and of the vegetation as a complex of biological attributes, is characterized as $\forall j \in \mathbf{J}$, $\forall \ell \in \boldsymbol{\Lambda}$, $\forall \beta \in \mathbf{B}$

$$\sum_{c\in\mathbf{C}\cup\mathbf{G}\cup\{h\}} \pi_{ic} q_{jc} = p_j, \tag{33}$$

$$\sum_{c\in\mathbf{S}\cup\mathbf{G}\cup\{h\}} \pi_{ic} q_{\ell c} = \sigma_{i\ell}(1 - \varphi), \tag{34}$$

$$\frac{dt_i}{d\varepsilon_i} = \sum_{\beta\in\mathbf{B}} \frac{\partial \nu_i}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_i}. \tag{35}$$

Remark 4

(i) The conditions presented are not only necessary but also sufficient from the assumptions on the functions. In the first equation, π_{ic} signifies a *hedonic shadow price* of attribute c acquired by utilizing inhabitant i's available time and his/her functionings. The left-hand side of the first equation is the sum of resident i's marginal evaluations of the tangible attributes embodied in one unit of a good, as well as of the gaseous attributes and heat released when the good is produced.

Notice that the first two formulas verify that any resident considers heat and gases as intangible characteristics emitted when consuming one unit of good j. The first conditions mean that the unit price of the good is equal to the sum of marginal contributions of attributes to his/her happiness through his/her functionings. The conditions assure a Pareto optimality for a quantity of each good, and they give a basis upon which goods resident i chooses to buy. He or she may not buy good j after due consideration of $\sum_{c\in\mathbf{G}\cup\{h\}}\pi_{ic}q_{jc}$, which may be smaller than p_j, because $\pi_{ic}q_{jc} < 0$ holds for some $c \in \mathbf{C}$ and

$$\sum_{c\in\mathbf{C}\cup\mathbf{G}\cup\{h\}} \pi_{ic}q_{jc} < \sum_{c\in\mathbf{C}} \pi_{ic}q_{jc} = p_j. \tag{36}$$

(ii) The second condition signifies that individual i's marginal evaluation of species as biological attributes by having the vegetation $V_{i\ell}$ in the area $A_{i\ell}$ of his/her land L_i, planted by landscape gardener ℓ is equal to his/her greening cost per cubic meter minus the refunded cost. The term $\sigma_\ell\varphi$ may be called a "urban warming (alleviating) subsidy," since it represents the reward according to the person's effort to plant trees in a part of his/her land. Many residents suffer from the fierce heat in summertime, if he/she feels very displeased by the boiling weather due to urban warming.

(iii) The condition about t_i shows that city dweller i's marginal evaluation of consuming a unit of electricity is a match for his/her marginal urban warming tax. Thus, a proper incentive is given to an inhabitant to have a part of his/her land planted with trees and flowers from motives of selfishness.

3.2 Offices and Manufacturers as Producers

Here I present the optimization by profit maximizing producers to supply one good with an optimal product quality to consumers. Let producer j (landscape gardener ℓ) produce good j (service ℓ) by using x_{j0} ($x_{\ell 0}$) as inputs, and the price of x_{j0} ($x_{\ell 0}$) is normalized to be one, with $\sum_{j\in\mathbf{J}} x_{j0} + \sum_{\ell\in\mathbf{\Lambda}} x_{\ell 0} \le \sum_{i\in\mathbf{N}} z_{i0}$. Then x_{j0} ($x_{\ell 0}$) is the amount of labor time that producer j (landscape gardener ℓ) uses as an input.

Denote $\nu_j(U(\varepsilon_j(\alpha_j), \varepsilon_{-j}))$ as producer j's damage function and $\alpha_j \ge 0$ is an abatement cost only known to j. Note that ν_j depends on the emissions of the rest of the city. It may be interpreted as an external cost to buy, for example, more efficient power saving air conditioners. Urban warming tax t_j is a function of ε_j and α_j represents producer j's effort to decrease emissions of heat and gases when producing and transporting $x_j = \sum_{i\in\mathbf{N}} x_{ij}$ units of good j. More examples to explain α_j are in order: producer j encourages his/her staff to commute by bicycle, to use hybrid or fuel cell cars or electric vehicles as delivery vans, and to choose energy-saving types of personal computers in its office.

It is generally accepted that midtown hotels and office buildings use more computers and air conditioners than residents and landscape gardeners. The former would emit more heat and gases into the urban atmosphere as to cool hotels and offices, as urban warming accelerates in the near future. As producers, offices offer services, manufacturers supply products. $\sigma_{j\ell}V_{j\ell}$ is producer j's greening cost, and $\varphi\sigma_{j\ell}V_{j\ell}$ is the refund to j.

Let $y_j = (x_{j0}, x_j, q_{j1}, \dots, q_{jC}, q_{jC+1}, \dots, q_{jC+G}, q_{jh})$ be producer j's input–output vector, then it produces a good j as an output to maximize its profit subject to the production function

$$\psi_j = \psi_j(x_{j0}, x_j, q_{j1}, \dots, q_{jC}, q_{jC+1}, \dots, q_{jC+G}, q_{jh}) \le 0, \tag{37}$$

where q_{jc}, $\forall c \in \mathbf{G}$, is the amount of heat or a gas emitted in the urban atmosphere when it produces one unit of good j, and $(q_{jC+1}, \dots, q_{jC+G}, q_{jh})$ is a vector of intangible attributes. Meanwhile, $(q_{j1}, \dots, q_{jC})$ is a vector of tangible characteristics embodied in one unit of good j. Any producer therefore jointly produces goods and attributes. The production function may not be convex, but the difficulties arising from non-convexities are not treated here, so an assumption is needed.

Assumption 4 For any $j \in \mathbf{N}$, ψ_j is convex and twice continuously differentiable with $\partial x_{j0}/\partial q_{jc} > 0$, $\forall c \in \mathbf{C} \cup \mathbf{G} \cup \{h\}$, $c \neq j'$. Furthermore, $x_j > 0$ implies $x_{j0} > 0$, and $\forall \eta \in \mathbf{R}_+$, $\{y_j | \psi_j(y_j) \le 0, x_{j0} \le \eta\}$ is compact.

It is assumed that π_{ic} and π_{ig} are truthful, since all goods are private goods or publicly provided goods such as gas, water, and electricity, for which the residents have to pay public utility charges. Hence, they cannot have them for free.

When $x_{ij} > 0$, p_j could be computed as $\sum_{c \in \mathbf{C} \cup \mathbf{G}\{h\}} \pi_{ic} q_{jc}$ from the FOCs, then the profit maximization problem for the producers is given by

$$\begin{aligned}\text{Max } P_j = {} & \sum_{i \in \mathbf{N}} \sum_{c \in \mathbf{C} \cup \mathbf{G} \cup \{h\}} \pi_{ic} q_{jc} x_{ij} - x_{j0} - \sigma_{j\ell} V_{j\ell} + \varphi \sigma_{j\ell} V_{j\ell} - t_j\bigl(\varepsilon_j(\alpha_j)\bigr) \\ & - \nu_j\bigl(U\bigl(\varepsilon_j(\alpha_j), \varepsilon_{-j}\bigr)\bigr).\end{aligned}$$

Here I make another assumption.

Assumption 5 For any $j \in \mathbf{J}$, t_j, e_j and t_j are convex, and they are twice continuously differentiable with $dt_j/d\varepsilon_j > 0$, $d\varepsilon_j/d\alpha_j < 0$, $\partial \nu_j/\partial U_\beta > 0$, $\partial U_\beta/\partial \varepsilon_j > 0$, $\forall \beta \in \mathbf{B}$.

In the presence of tangible and intangible attributes, the necessary conditions for Pareto optimal product quality in terms of tangible and intangible attributes are for any producer $j \in \mathbf{J}$: $\forall c \in \mathbf{C}$, $c \neq j'$, $\forall g \in \mathbf{G}$, $g \neq j'$, $h \neq j'$

$$\sum_{i \in \mathbf{N}} \pi_{ic} x_{ij} = \frac{\partial x_{j0}}{\partial q_{jc}}, \tag{38}$$

$$\sum_{i \in \mathbf{N}} \pi_{ig} x_{ij} = \frac{\partial x_{j0}}{\partial q_{jg}} + \sum_{\beta \in \mathbf{B}} \frac{\partial \nu_j}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_j} x_j, \tag{39}$$

$$\sum_{i \in \mathbf{N}} \pi_{ih} x_{ij} = \frac{\partial x_{j0}}{\partial q_{jh}} + \sum_{\beta \in \mathbf{B}} \frac{\partial \nu_j}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_j} x_j, \tag{40}$$

$$\frac{dt_j}{d\varepsilon_j} = \sum_{\beta \in \mathbf{B}} \frac{\partial \nu_j}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_j}. \tag{41}$$

Remark 5

(i) The first equation establishes a Pareto optimality for an amount of each attribute and determines a vector of optimal tangible characteristics embodied in the good supplied by producer j. The L.H.S. of the first equation is the marginal revenue which is the aggregate of the residents' marginal evaluations of a change in an attribute embedded in x_j. Its R.H.S. is the marginal cost in terms of labor time to produce q_{jc}. $\sum_i \pi_{ic} x_{ij}$ is the marginal social value of good j, which is the sum of the personal evaluations of a change in an attribute when the quantity of good x_j is produced.

(ii) The third equation shows a Pareto optimal quantity of each gas as an intangible attribute. The L.H.S. is the social value of heat, and the R.H.S. consists of the terms: the first two term is the marginal cost in terms of x_{j0}, and the second term means the marginal climatic damage to emit heat. t_j is the urban warming tax of producer j.

3.3 Landscape Gardeners

Next, efficiency conditions for landscape gardeners are derived. Their job is to plant trees in areas of lands that residents and producers possess, so they are exempted from the obligation of greening a part of their own lands in our model. They supply plants as biological, and thus, tangible attributes, as well as intangible attributes.

Denote $\nu_\ell(U(\varepsilon_\ell(\alpha_\ell), \varepsilon_{-\ell}))$ as landscape gardener ℓ's heat abatement cost and $\alpha_\ell > 0$ is a parameter only known to gardener ℓ. Landscape gardeners can attempt to serve green areas to residents and producers by using a new technology which does not emit too much heat. Let $\mathbf{N}_\ell$ be the set of residents who commission landscape gardener ℓ to plant trees and $\mathbf{J}_\ell$ be the set of producers which ask gardener ℓ to plant trees $V_\ell = \sum_{i \in \mathbf{N}_\ell} V_{i\ell} + \sum_{j \in \mathbf{J}_\ell} V_{j\ell}$.

Any gardener provides a green area to residents and producers to maximize his/her profit subject to the production function

$$\psi_\ell = \psi_\ell(x_{\ell 0}, L_\ell, V_\ell, q_{\ell C+1}, \ldots, q_{\ell C+G}, q_{\ell C+G+1}, \ldots, q_{\ell C+G+S}, q_{\ell h}). \tag{42}$$

Hence, each gardener solves the optimization problem

$$\begin{aligned} \text{Max } P_\ell = & \sum_{i \in \mathbf{N}_\ell} \sum_{c \in \mathbf{S} \cup \mathbf{G} \cup \{h\}} \pi_{ic} q_{\ell c} V_{i\ell} + \sum_{j \in \mathbf{J}_\ell} \sigma_{j\ell} V_{j\ell} - x_{\ell 0} + t_\ell\big(\varepsilon_\ell(\alpha_\ell)\big) \\ & + \nu_\ell\big(U\big(\varepsilon_\ell(\alpha_\ell), \varepsilon_{-\ell}\big)\big). \end{aligned}$$

Two assumptions are needed for this maximization problem.

Assumption 6 For any $\ell \in \boldsymbol{\Lambda}$, ψ_ℓ is convex and twice continuously differentiable with $\partial x_{\ell 0}/\partial q_{\ell c} > 0$, $\forall c \in \mathbf{S} \cup \mathbf{G} \cup \{h\}$, $c \neq \ell'$. Furthermore, $A_\ell > 0$ implies $x_{\ell 0} > 0$ and $L_\ell > 0$, and $\forall \eta \in \mathbf{R}_+$, $\{y_\ell | \psi_\ell(y_\ell) \leq 0, x_{\ell 0} \leq \eta, L_\ell \leq \eta\}$ is compact.

Assumption 7 For any $\ell \in \boldsymbol{\Lambda}$, t_ℓ, ε_ℓ and ν_ℓ are convex and twice continuously differentiable with $dt_\ell/d\alpha_\ell < 0$, $d\varepsilon_\ell/d\alpha_\ell < 0$, $\partial\nu_\ell/\partial U_\beta < 0$ and $\partial U_\beta/\partial\varepsilon_\ell < 0$, $\forall\beta \in \mathbf{B}$.

The FOCs are the following. For any landscape gardener $\ell \in \Lambda$, necessary conditions for Pareto optimal quality of planted vegetation as a complex of biological attributes are $\forall s \in \mathbf{S}$, $s \neq \ell'$, $\forall g \in \mathbf{G}$, $g \neq \ell'$

$$\sum_{i\in\mathbf{N}_\ell} \pi_{is} V_{i\ell} + \sum_{j\in\mathbf{J}_\ell} \sigma_{j\ell} \frac{\partial V_{j\ell}}{\partial q_{\ell s}} = \frac{\partial x_{\ell 0}}{\partial q_{\ell s}}, \tag{43}$$

$$\sum_{i\in\mathbf{N}} \pi_{ig} V_{i\ell} + \sum_{j\in\mathbf{J}_\ell} \sigma_{j\ell} \frac{\partial V_{j\ell}}{\partial q_{\ell g}} = \frac{\partial x_{\ell 0}}{\partial q_{\ell g}} + \sum_{\beta\in\mathbf{B}} \frac{\partial \nu_\ell}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_\ell} V_{i\ell}, \tag{44}$$

$$\sum_{i\in\mathbf{N}} \pi_{ih} V_{i\ell} + \sum_{j\in\mathbf{J}_\ell} \sigma_{j\ell} \frac{\partial V_{j\ell}}{\partial q_{\ell h}} = \frac{\partial x_{\ell 0}}{\partial q_{\ell h}} + \sum_{\beta\in\mathbf{B}} \frac{\partial \nu_\ell}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_\ell} V_{i\ell}, \tag{45}$$

$$\frac{dt_\ell}{d\varepsilon_\ell} = \sum_{\beta\in\mathbf{B}} \frac{\partial \nu_\ell}{\partial U_\beta} \frac{\partial U_\beta}{\partial \varepsilon_\ell}. \tag{46}$$

Remark 6 The equations establish a Pareto optimality for the amount of each attribute and determine a vector of optimal quality characteristics that any gardener can supply its biological product to residents and producers. In the second and third equations, the R.H.S. are composed of two terms: the first is the marginal cost of the labor to supply one unit of plant as a biological attribute and the second term is gardener ℓ's marginal contribution to urban warming by emitting heat. The L.H.S. of the equations signify the marginal social value in terms of the labor time, where the marginal social value is the sum of the evaluations of a change in each biological characteristic.

3.4 Urban Warming Tax and Subsidy

This section proposes an urban warming (alleviating) tax/subsidy scheme. For that purpose, define the monetary damages due to urban warming as the sum of costs to deal with public damages: $\Phi \equiv \sum_{\beta\in\mathbf{B}} D_\beta(U)$. Examples are the economic losses due to the deaths of people and the inundation of subway stations. Φ embraces houses flooded above floor level or up to the floorboards at some blocks and domesticated animals due to heat waves. The total cost to cope with private damages: $\Psi \equiv \sum_{e\in\mathbf{E}} \nu_e(U)$ includes the costs of cooling installations in stockyards and of increasing water consumption due to cooling them. The social damage due to urban warming therefore is the following. Hence, our problem of social cost to be minimized is

$$\text{Min}\ \{\Phi + \Psi\}.$$

An assumption is imposed.

Assumption 8 D_β is concave and twice continuously differentiable with $\partial D_\beta/\partial U_\beta > 0$ and $\partial^2 D_\beta/\partial U_\beta^2 > 0$, $\forall\beta \in \mathbf{B}$.

Suppose that emissions of heat and gases are integrated to be represented by one argument $\varepsilon_e \equiv z_{eg} \equiv z_{eh}$. We seek for the socially optimal amount of heat emissions. We have

$$\sum_{\beta\in\mathbf{B}} \frac{\partial D_\beta(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right)\frac{d\varepsilon_e}{da_e}\,da_e + \sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}}\sum_{\beta\in\mathbf{B}} \frac{\partial \nu_e(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right)\frac{d\varepsilon_e}{da_e}\,da_e = 0. \quad (47)$$

Here is the main result.

Theorem 1 *Urban Warming Tax is a mechanism of the form à la Groves represented by,* $\forall e \in \mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}$, $\forall\beta\in\mathbf{B}$:

$$t_e(\varepsilon_e) = -\int_0^{\varepsilon_e} \frac{\partial \nu_e(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right)\frac{d\varepsilon_e}{da_e}\,da_e + I_e(\varepsilon_{-e}),$$

where $I_e(\varepsilon_{-e})$ *is a constant of integration independent of* ε_e.[6]

Proof By adopting the Laffont (1982) differential method, one observes from the FOCs above that

$$\sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}} \left\{\frac{\partial \nu_e(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right) + \frac{dt_e(\varepsilon_e)}{d\varepsilon_e}\right\}\frac{d\varepsilon_e}{da_e}\,da_e = 0, \quad \forall\beta\in\mathbf{B}. \quad (48)$$

Integrating this yields

$$\sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}} t_e(\varepsilon_e) = -\int_0^{\varepsilon_e}\left\{\sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}} \frac{\partial \nu_e(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right)\frac{d\varepsilon_e}{da_e}\,da_e + I_e(\varepsilon_{-e})\right\}. \quad (49)$$

Removing the summation from Eq. (49) leads the statement of Theorem 1. Suppose that there exists a unique solution to the above problem of social cost minimization. As we aim to find the urban warming tax corresponding to the social optimum, substituting Eq. (47) in Eq. (49) gives

$$\sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}} t_e(\varepsilon_e) = \sum_{\beta\in\mathbf{B}}\int_0^{\varepsilon_e} \frac{\partial D_\beta(U_\beta)}{\partial U_\beta}\left(\frac{\partial U_\beta}{\partial \varepsilon_e}\right)\frac{d\varepsilon_e}{da_e}\,da_e + I, \quad (50)$$

where I is a constant of integration.

Consequently, the sum of *urban warming tax* is represented by

$$\sum_{e\in\mathbf{N}\cup\mathbf{J}\cup\boldsymbol{\Lambda}} t_e(\varepsilon_e) = \sum_{\beta\in\mathbf{B}} D_\beta(T_\beta) + S, \quad (51)$$

where S is a constant of integration. □

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[6] See Groves (1976), Green and Laffont (1979), and Myles (1995).

In the above equation, S interpreted as the amount of subsidy can be refunded to those collaborating on tree-planting plus the costs of public utilities. Let me propose therefore the following.

Claim 1 *Urban Warming Subsidies are given to the collaborators who have planted trees and vegetation to cool their city. We have*

$$S = \sum_{i \in \mathbf{N}} \sum_{\ell \in \boldsymbol{\Lambda}} \varphi_i \sigma_{i\ell} V_{i\ell} + \sum_{j \in \mathbf{J}} \sum_{\ell \in \boldsymbol{\Lambda}} \varphi_j \sigma_j V_j .$$

Subsidies are, respectively, given to residents, offices, and manufacturers, since they have cooperated in their efforts to have trees and flowers planted in the required percentage of their land lots. An important feature is that the tax levied may fully cover social damages, and the part of the tax paid S can be redistributed to the contributors in order to cool their city.

4 Final Remarks

This paper has analyzed urban warming and heat island in the theory of attributes/functionings à la Gorman–Lancaster–Sen that I proposed in Sato (2008). My discussion has proceeded on the premise that the urban atmosphere is a composite of intangible characteristics including heat. This paper has extended Sato (2006, 2008) to combine global and urban warming and the urban heat island in an integrated energy balance model. Our model incorporates landscape gardeners who are commissioned by residents and producers. Then an optimization by each agent is solved to derive optimality conditions in the attributes/functionings theory. Residents are happiness maximizers and producers and landscape gardeners are profit maximizers, and the metropolitan government is a social cost minimizer. An urban warming tax/subsidy scheme has been proposed; the subsidy is given to contributors of cooling the city, by planting. This paper has also shown that the change of composition of tangible attributes in the goods changes that of intangible attributes such as heat and GHGs in the urban atmosphere.

As a recent technological innovation, let me introduce the *photocatalyst of titanic oxide* (TiO_2) to cool-down structures. This method clarified that water can be decomposed only by photo energy and titanic oxide. In 2006, an experiment was conducted on a roof-top (10 m^2) of a building, which was covered by a water film coated with titanic oxide. This test proved that this film of water could cool the building and showed that the temperature decreased by 30 °C. More precisely, that experiment verified that the mercury stood at 60 °C without the photocatalyst and water sprinkling, whereas the temperature was 30 °C with both. The maximum temperature difference between the two cases amounted to 30 degrees. The indoor temperature at that time was 15 °C, which resulted in the cost reduction of air conditioning. The photocatalyst of titanic oxide on glass on the sides of the buildings can cool down their structures and save on the cost of air conditioning. The cooling effect and cost effectiveness could be augmented by utilizing rainwater and by coating windows with titanic oxide. Currently, the size of the market for this technology amounts to 0.4 billion dollars, and it is anticipated to augment to 8.3 billion dollars in the future. In 2006 also, a new variety of photocatalyst has been discovered; applying the radiant energy resolves water to raise hydrogen, which is well known as an ultimate clean energy. A cocatalyst added to yellow powder as a composite of nitrogen gallium and zinc oxide becomes a catalyst which reacts by radiant energy. The resolution efficiency of water is about ten times higher than the former one and there is still room for enhancing further efficiency. In 2009, a new system of photocatalysis, the *titania photocatalyst sheet* was invented.

In 2008, it was proposed that a *Sierpinsi manifold* could be an alternative to the leaves of trees to cool down warming cities by discovering the fractal similarity of the former and the latter. Seen from one direction, a Sierpinski manifold is a plane, whereas it is three-dimensional when it is seen from the other. By selecting a tree, it has been observed that its fractal is two-dimensional, although it has a three-dimensional structure. Samples of the Sierpinski manifold were made and placed on the roof of a building in a central business district in a city and obtained the desired result of cooling down by mitigating solar heat and light under their structure. It was verified that the trees have two fractal dimensions which are the same as the *Sierpinski Tetrahedron*. A roof as a complex of many Sierpinski tetrahedrons could be a substitute to trees, in terms of an efficient method to combat against urban warming and heat island. Some experiments were conducted by constructing roofs made by Sierpinski tetrahedrons and showed that the temperature beneath these roofs was lower than that measured on a road without the roofs. What is important is that maintenance and waterworks are not necessary for these manifolds, which could save on the cost of water. Different from trees, water is not necessary and just the management of roofs is needed.[7]

As in Green et al. (2007), we must challenge ourselves to work towards stabilizing the urban climate by reducing anthropogenic heat stocks in regions of cities at all costs. We cannot foresee what will happen in the near future, so we had better take every precaution against urban warming and heat islands.

Competing Interests

The author declares that they have no competing interests.

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[7] See Sato (2014) for water-related technological innovations.

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