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Quality-adjusted productivity gain in the propagation of innovation

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Abstract

We introduce a class of production function whose inputs and outputs constitute multiples of quality and quantity. Under the efficiency unit approach, we precisely reduce innovation regarding qualitative and quantitative improvements of production to the measurement of quality-adjusted productivity gain. We then consider a system of compatible unit cost functions inclusive of such productivity improvements in any industry, for which we can solve for the ex-post equilibrium to examine the technological structural propagation. In this way, we can evaluate any given innovation with respect to its social welfare gain. We use this framework of multi-industry multi-factor production to study effective industry-wise research and development investment allocations.

JEL classification: D24; O33; O41

Keywords: Productivity; Innovation; Efficiency unit; R&D policy; General equilibrium

1 Background

Research and development (R&D) is considered the central driving force of national competitive advantage. Effectively using limited R&D resources is an important agenda when promoting and evaluating national level R&D programs Lee et al. (2009). In some cases, a large portion of national R&D resources are distributed to target technologies under the "selective focus and contraction strategy" Lee and Song (2007). However, we believe that this policy could impede the harmonious and sound development of the economy, because of mutual interdependencies in current and potential industrial production technologies. With this in mind, we are interested in investigating what allocations of public R&D investment could promote effective innovations and gain more welfare.

Contrary to R&D investment, innovation (the consequence of R&D) has been postulated as an intermediate stage of gaining welfare, rather than a measurement in terms of monetary value. Innovation is commonly defined as the implementation of a new or significantly improved product (good or service), process, marketing method, or organizational method in business practices, workplace organization, or external relations OECD/Eurostat (2005). Most work on innovation (as reviewed in Hall (2011)) has used surveys based on a version of this definition, typically using dummy variables that represent a product and/or process innovation (e.g., Griffith et al. (2006)), innovative sales shares (e.g., van Leeuwen and Klomp (2006)), or patent counts (e.g., Crépon et al. (1998)) as proxies to explain the growth of productivity.

Productivity growth fills the gap between growths in the quantitative inputs and outputs of production. Hence, a productivity growth signifies welfare increase that can be attributed to innovation. In other words, the productivity growth (or gain) can be used to evaluate the economic significance of an innovation that occurred within some interval of time. Additionally, because R&D is considered to inflate productivity, many researchers have investigated the connection between R&D investment (including R&D capital stock) and productivity growth (among others Griliches (1994), Hall and Mairesse (1995), Sakurai et al. (1997), Kuroda and Nomura (2004), Parisi et al. (2006), Coccia (2009), and Hall et al. (2009)).

This study also takes the position that R&D's direct achievement is the gain in productivity. We are interested in the consequential influence of R&D investment, rather than the mechanism behind R&D's promotion of innovation. Therefore, we directly use productivity gain as a measure of innovation, and then relate R&D investments with that measure.[1] More importantly, in contrast to previous research, we are concerned with the economy-wide propagation of innovation. Productivity gain reduces the marginal cost of production, and the corresponding price change influences the selection of technologies (substitution of inputs) for other industries, because of technological interdependencies between industrial sectors. A system of unit cost functions (which are compatible with constant returns to scale) can be used to model the economy-wide technological structural equilibrium and the propagation triggered by any exogenous productivity gain.

At the same time, we are also concerned with qualitative changes in the inputs and outputs of production. In this regard, we take the efficiency unit approach Hulten (1992), which incorporates quality in terms of quantity. The underlying idea of the efficiency unit approach is to consider everything in terms of efficiency units (*effective* quantities), using the marginal rate of substitution (MRS) for the target compared with the standard-quality commodity. Because economic efficiency equalizes MRS and price ratios, the qualities (the MRS with respect to the standard) and prices used to differentiate the quality of (perfect substitute) commodities become proportional Gordon (1990) under the static equilibrium for a certain technology state (see Fig. 1). However, innovation breaks this proportionality. Accordingly, we use this proportional disparity to measure innovation.

We can measure proportionality shifts using hedonic regression, where the set of observed prices are regressed against the Lancasterian attributes Lancaster (1966) of goods. This is a commonly used approach for estimating quality-adjusted consumer price indices (CPI).[2] Proportionality shifts can be measured via the price change (in the form of a deflator) of a quality-standardized commodity. Because such a deflator includes qualitative and quantitative improvements, we call it a quality-adjusted deflator. Monetary accounts of different periods are typically realized using deflators that only consider the price change. The quality-adjusted deflator considers the price and quality changes.[3]

For simplicity, we assume that any production is subject to constant returns to scale in effective quantities, whereas any assessment of quality (MRS) is universal, meaning

[1] We quantify the significance of innovation using the gain in total factor productivity (e.g., Fukao et al. (2007), Park (2012), and Sheng and Song (2013)), ignoring various associated spillover effects (e.g., Dietzenbacher (2000), Hanel (2000), and Jacobs et al. (2002)).

[2] Triplett Triplett (2006) is an encompassing review of this subject. Jonker Jonker (2002) compares the hedonic with the discrete choice approach. Nakano and Nishimura Nakano and Nishimura (2012) provides a welfare compatible assessment of qualitative change, based on the discrete choice approach.

[3] CPI for cameras and personal computers are quality-adjusted via the hedonic method and are used in recent official price deflators Statistics Japan (). Otherwise, deflators are not quality-adjusted.

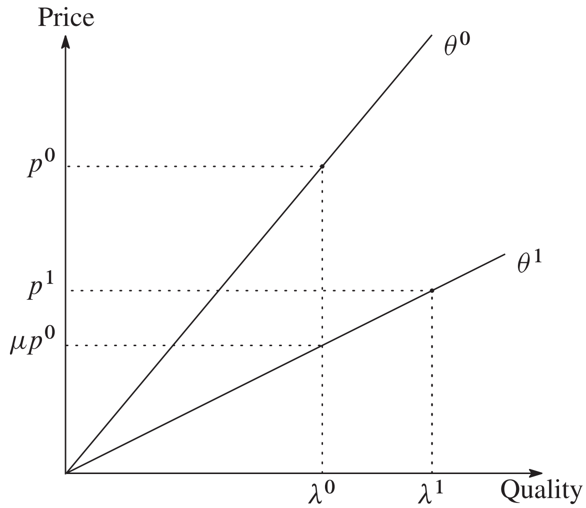

Fig. 1 Proportionality shift

that the same quality-adjusted deflators are applicable to any production or consumer. Then, the combined qualitative and quantitative innovation exclusively within an industry can be measured by applying quality-adjusted deflators to both inputs and outputs of production. We call this *local* innovation measure the quality-adjusted productivity gain of an industry.[4] Naturally, we relate industry-wise R&D investments with industry-wise quality-adjusted productivity gains. We use quality-adjusted unit cost functions (which map the costs of effective unit outputs) to evaluate the propagative effects of industry-wise R&D investments with respect to the gain in social welfare.

In Section 2, we introduce the concept of quality-adjusted productivity, which reflects the innovation that is exclusive to an industry (i.e., local innovation). This is closely related to the industry-wise R&D. In Section 3, we introduce the concept of structural propagation, which is initiated by the introduction of innovation involving qualitative and quantitative improvements, and connect the ex-post equilibrium state with social welfare. In addition, R&D investment allocation optimization is demonstrated via a small prototype model. Section 4 contains our concluding remarks.

2 Measurement

2.1 Illustrative example

The following example demonstrates the essence of our task. We assume that a new type of paint has been invented, which means that car manufacturers can use less paint during production. Additionally, they can also produce cars with less metal because the new paint enhances the strength of the materials on which it is applied. The new paint is easier to apply, requiring less labor in the manufacturing process. Moreover, consumers can derive more utility from the cars because the new paint looks good. As a result, the new paint affects technology selections (i.e., paint–metal and paint–labor substitutions), the quality (i.e., an attractive car), and the cost of the car.

[4]Note that ordinary productivity gain may include innovative contributions embodied in the factor inputs, which we call *foreign* innovations. The basic idea is to exclude all the indirect (foreign) contributions from the gross measurement of innovation and focus on the internally established portion.

The first assumption we make is that any two commodities of the same kind with different qualities are perfect substitutes. Suppose that there are two cans of paint named Sirius and Vega, such that 1 L of Sirius is as effective as 2 L of Vega. Then, the MRS of Sirius against Vega is 2. We can use this MRS as the measure of quality for Sirius, relative to the reference standard paint Vega. The second assumption is that this quality measure (MRS) is universal, meaning that the same MRS is applicable to any industry or individual who is willing to consume the substituting commodities. The third assumption is that quality and price are proportional in the static equilibrium. If we keep time stationary to eliminate any innovations, coexisting perfect substitutes must have price ratios equal to the MRS. Hence, Sirius must be twice as expensive as Vega if these paints were to coexist in the market. Conversely, the more cost-efficient model must dominate the market if they are not proportional.

Table 1a shows the inputs and outputs of the car industry. The car industry produces a variety of cars. Suppose we observe that a car named Fox is 1.5 times as expensive as a reference standard car named Rabbit, which costs 1130 thousand yens (Kyens, hereafter). Because the price of a car is proportional to its quality, Fox's quality must be 1.5 with respect to Rabbit's. Dividing the price of any car by its quality produces a common constant, which we call the standardized price. In this case, the standardized price of a car is 1130 Kyens, where Rabbit is the standard. In the same manner, the standardized prices of paint, metal, and labor are 25 Kyens per liter, 500 Kyens per ton, and 350 Kyens per month, respectively. These values are based on the standard commodities, namely, Vega, steel, and engineer. Observing the input–output accounts for the car industry, we have the standard input–output quantities based on standard commodities. We assume that standardized quantities are subject to the Cobb–Douglas production function, so we obtain the benchmark absolute productivity (0.419).[5]

Suppose that, after some time, a new paint called Capella enters the market (Table 1b). Capella's quality (MRS with respect to Vega) is the same as Sirius's (2), and it costs 32 Kyen/liter (less than twice Vega's). In this case, Vega and Sirius are eliminated and only Capella (and paints that are proportional to Capella) remains in the market. Note that the standardized price of paint is now 16 Kyen/liter. This means that we may now acquire 1 L of Vega equivalent paint for 16 Kyens. This standardized price change in paint affects the standardized input–output quantities for producing one standardized car, which can be calculated using the production function. We now have the new monetary inputs that combine to a unit cost of 974 Kyens for producing one Rabbit.[6]

We may then consider the industry's productivity using the per-yen standardized output (measured by the number of Rabbits). Suppose that we observe a very attractive car (quality level of 1.2) called Weasel, which costs 1169 Kyens. Because the ex-post standardized price of a car is $1169/1.2 = 974$ Kyens, Weasel is Rabbit-proportionate. The productivity gain can be calculated by the ratio of reciprocals of the standardized prices for two periods, i.e., $(1/974)/(1/1130) = 1.160$. However, because the standardized price of a car is reduced to the standardized unit cost, we know that there have been no innovations within the car industry. A relevant measure of innovation should not change in this case. When assessing the productivity gain for only the car industry, we should eliminate

[5] The absolute productivity, z, should satisfy $1 = z(15)^{0.332}(0.6)^{0.265}(1.3)^{0.403}$, because the cost shares coincide with the output elasticities of the underlying Cobb–Douglas production function.

[6] Note that quantities are measured by the standard goods in Table 1a–c. Input quantity of paint in Table 1b, for example, is hence $0.332 \times 974/25 = 12.94$ L (of Vega).

Table 1 Car paint example

	Output		Inputs	
	Car	Paint	Metal	Labor
(a) Benchmark input-output accounts for the car industry.				
Price (standard)	1130	25	500	350
Quality (standard)	1	1	1	1
Quantity	**1**	**15**	**0.6**	**1.3**
Unit cost (standard)	**1130**	**375**	**300**	**455**
Cost share	**1**	**0.332**	**0.265**	**0.403**
Productivity		**0.419**		
(b) Input-output accounts for the car industry ex-post of paint innovation.				
Price (sample)	1169	32	500	350
Quality (sample)	1.2	2	1	1
Price (standardized)	974	16	500	350
Quantity	**1**	**12.94**	**0.517**	**1.121**
Unit cost (standardized)	**974**	**323**	**259**	**392**
Productivity		**0.486 (gain: 1.160)**		
Productivity (quality-adjusted)		**0.419 (gain: 1.000)**		
(c) Input-output accounts for the car industry ex-post of paint and car innovations.				
Price (sample)	1300	32	500	350
Quality (sample)	1.65	2	1	1
Price (standardized)	788	16	500	350
Quantity	**1.237**	**12.94**	**0.517**	**1.121**
Unit Cost (standardized)	**974**	**323**	**259**	**392**
Productivity		**0.602 (gain: 1.434)**		
Productivity (quality-adjusted)		**0.519 (gain: 1.237)**		

any contribution from the factor inputs. In this regard, we should only consider the per-yen output change between the standardized unit cost and the standardized price, which is $(1/974)/(1/974) = 1.000$. We distinguish the productivity gain that reflects the local innovation of an industry (in this case, the car industry) from the ordinary productivity gain estimated via the standardized price change of the output.[7]

Alternatively, suppose that there is some local innovation in the car industry, and a new car called Sable enters the market (Table 1c). A Sable costs 1300 Kyens, and its quality (MRS with respect to a Rabbit) is 1.65. The standardized price of a car is now $1300/1.65 = 788$ Kyens per Rabbit, and Rabbit and Fox must both exit the market. Only the Sable-proportional cars can exist in the ex-post market. The productivity gain is then $(1/788)/(1/1300) = 1.434$ with respect to the benchmark, but obviously this number is inclusive of the contribution of the new paint, Sirius. The relevant productivity gain is $(1/788)/(1/974) = 1.237$, which should reflect the innovation from the Rabbit-proportional to Sable-proportional cars. We call this measure of the local innovation the quality-adjusted productivity gain, because we use standardized figures for both the input and output sides of production. Furthermore, note that this measure should reflect the outcome of the R&D in the car industry. This is how we estimate quality-adjusted productivities for different industries in this study.

[7] The ex-post absolute productivity satisfies $1 = z(12.94)^{0.332}(0.517)^{0.265}(1.21)^{0.403}$, whereas the absolute quality-adjusted productivity satisfies $1 = z(323/16)^{0.332}(0.517)^{0.265}(1.21)^{0.403}$.

2.2 Quality-adjusted productivity

We now introduce a class of production function that connects inputs and outputs accounted in effective quantities. We denote the effective output in units of standard commodity by λy, where y denotes the nominal output quantities, and λ denotes the quality measure, i.e., the MRS of the output commodity with respect to the standard commodity. There are $n+1$ kinds of inputs. The nominal quantity of input commodity i is x_i, and the quality measure (i.e., the MRS of the input commodity with respect to the standard) is λ_i. Hence, the effective input is $\lambda_i x_i$. The production function of an industry (the index j is omitted) is

$$\lambda y = zf\left(\lambda_0 x_0, \lambda_1 x_1, \ldots, \lambda_n x_n\right) = zf\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right), \tag{1}$$

where the dot operator represents element-wise multiplication. Here, z denotes the absolute productivity, which reflects the technology level of the industry in question. Any R&D investment allocated to this industry is assumed to affect z in an exclusive manner. Accordingly, we assume that the frame (i.e., the parameters) of the remaining part $f(\cdots)$ in the above formula does not change over time.[8]

Taking the log and time derivative, we have

$$\frac{\dot{\lambda}}{\lambda} + \frac{\dot{y}}{y} = \frac{\dot{z}}{z} + \sum_{j=0}^{n} \left(\frac{\partial f\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right)}{\partial \lambda_i x_i} \frac{\lambda_i x_i}{f\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right)} \right) \frac{\lambda_i \dot{x}_i + \dot{\lambda}_i x_i}{\lambda_i x_i}. \tag{2}$$

The term in parenthesis is the cost share, which we denote by α_i. This is obvious from the following analysis. Given the instantaneous price of the output p and inputs $\mathbf{p} = (p_0, p_1, \ldots, p_n)$, a marginal firm's problem is

$$\max_{y,\mathbf{x}} py - \mathbf{p}\mathbf{x}' \quad \text{s. t.} \quad \lambda y = zf\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right).$$

The first order conditions for this problem are

$$\frac{pz}{\lambda} \frac{\partial f(\boldsymbol{\lambda} \cdot \mathbf{x})}{\partial \lambda_i x_i} \lambda_i = p_i.$$

Hence, we have

$$\frac{\partial f\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right)}{\partial \lambda_i x_i} \frac{\lambda_i x_i}{f\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right)} = \frac{p_i x_i}{py} \equiv \alpha_i. \tag{3}$$

The first order conditions for this problem are

$$\Delta \ln z = \Delta \ln \lambda y - \sum_{i=0}^{n} \alpha_i \Delta \ln \lambda_i x_i, \tag{4}$$

where Δ indicates the observed differences between two periods. This measurement of local innovation can be viewed as the quality-adjusted productivity growth. We may obtain ordinary productivity growth by ignoring the differences in the observed qualities, i.e., $\boldsymbol{\lambda} = \mathbf{1}$.

2.3 Measurement of quality-adjusted productivity growth

From a measurement perspective, quality-adjusted productivity growth requires inter-temporal differences of input and output accounts in effective quantities, according to (4). Here, we consider how the inter-temporal growth of quality-adjusted productivity can actually be measured. For now, we focus on the growth of output,

[8] Without spillover, R&D investment can only promote local innovation, so we use z as the measure of local innovation.

$$\Delta \ln \lambda y = \ln \lambda^1 y^1 - \ln \lambda^0 y^0. \tag{5}$$

Superscripts indicate that variables are either for the benchmark period (0) or the ex-post period (1).

In each period, the qualities and prices of perfect substitutes (commodities that are produced by the same industrial category) must be proportionate; all coexisting perfect substitutes must have price ratios equal to the MRS. For the two periods,

$$p^1 = \theta^1 \lambda^1, \quad p^0 = \theta^0 \lambda^0. \tag{6}$$

where θ denotes the proportionality, or the price of quality, which reflects the state of technology of a given period. Figure 1 illustrates this price-quality proportionality for two different periods. Note that innovations cause the proportionality to change over time. We can measure the extent of this change as the ratio between the slopes, i.e., $\mu = \theta^1/\theta^0$. Combining (6) and (5), and because we know from Fig. 1 that $\Delta \ln \lambda = \Delta \ln p - \ln \mu$, we have the identity

$$\Delta \ln \lambda y = \Delta \ln py + (\Delta \ln \lambda - \Delta \ln p) = \Delta \ln Y - \ln \mu,$$

where $Y = py$ denotes the monetary output for this industry, which is available from the input–output account. μ is the quality-adjusted deflator, which is typically estimated using hedonic regression or other methods (as outlined in Appendix A).[9] In a similar manner, we can derive a formula for measuring quality-adjusted productivity growth (instead of (4)). That is,

$$\Delta \ln z = (\Delta \ln Y - \ln \mu) - \sum_{i=0}^{n} \alpha_i (\Delta \ln X_i - \ln \mu_i) \tag{7}$$

where X_i is the monetary input from the ith industry and μ_i is the quality-adjusted deflator.

3 Propagation

3.1 Local innovation and welfare gain

We assume that the production function (1) has a constant returns to scale (CRS) property with respect to the effective quantities of inputs and outputs. Then, the CRS will hold for nominal quantities of inputs and outputs, namely, x_i and y. So, the unit cost of producing a nominal quantity, ρ, can be expressed using some strictly concave function of benchmark factor prices, $\mathbf{p}$. That is,

$$\rho = \frac{\lambda}{z} h(\mathbf{p}; \boldsymbol{\lambda}).$$

Local innovation enhances the productivity z, while altering quality λ. So, we can express the temporal change in the local unit cost exclusively caused by local innovation as

$$\Delta \ln \rho = \Delta \ln \lambda - \Delta \ln z. \tag{8}$$

Let us further standardize the parameters, z and λ, ex-post of local innovation, with respect to the benchmark. Rescaling the variables to set the benchmark values of z and λ to unity, the benchmark value of ρ must also be 1 (now, ρ is a deflator). Then, we can

[9] Note that we only obtain ordinary productivity if we use the deflator that measures changes in the quantity-weighted nominal prices.

use λ, z, and ρ to denote the benchmark-standardized ex-post parameters so that (8) is reduced, that is,

$$\lambda/\rho = z. \tag{9}$$

The interpretation of (9) is quite simple. The marginal quality of local deflation (thus, the proportionality) is equal to the gain in productivity z defined in (1). Moreover, from a measurement perspective, this productivity gain must be equal to the quality-adjusted productivity gain that reflects the same local innovation. Considering the previous car paint example, with quality gain $\lambda = 1.61/1$ and local deflation $\rho = 1,300/974$, (9) can be used to derive the correct quality-adjusted productivity gain, i.e., $z = (1.61/1)/(1,300/974) = 1.27$.

The ex-post problem of a marginal firm is then

$$\max_{y,\mathbf{x}} \ \rho\pi y - (\boldsymbol{\rho} \cdot \boldsymbol{\pi})\,\mathbf{x}' \quad \text{s. t.} \quad \lambda y = zf\left(\boldsymbol{\lambda} \cdot \mathbf{x}\right), \tag{10}$$

given that the ex-post price is denoted by $\rho_i\pi_i$ for the ith commodity, where $\boldsymbol{\pi}$ denotes the underlying price.[10] We can show that $\boldsymbol{\pi}$ only depends on the given bundle of productivity gains, $\mathbf{z}$. To do so, we introduce new variables, $\mathbf{w} = (w_0, w_1, \cdots, w_n) = (\lambda_0 x_0/z_0, \lambda_1 x_1/z_1, \cdots, \lambda_n x_n/z_n)$ and $u = \lambda y/z$, and rewrite (10) as

$$\max_{u,\mathbf{w}} \ \pi u - \boldsymbol{\pi}\mathbf{w}' \quad \text{s. t.} \quad u = f\left(\mathbf{z} \cdot \mathbf{w}\right).$$

Given that $f(\cdots)$ is homogeneous with degree one, π must be a function of the factor prices for $\mathbf{w}$ (in this case, $\boldsymbol{\pi}/\mathbf{z} = (\pi_0/z_0, \pi_1/z_1, \ldots, \pi_n/z_n)$). Then, we know that $\boldsymbol{\pi}$, except for π_0, is the solution to the system of unit cost functions $h_i(\cdots)$ and only depends on $\mathbf{z}$ viz.,

$$\boldsymbol{\pi} = (\pi_0, \pi_1, \cdots, \pi_n) = (\pi_0, h_1(\boldsymbol{\pi}/\mathbf{z}), \cdots, h_n(\boldsymbol{\pi}/\mathbf{z})) = (\pi_0, \mathbf{h}(\boldsymbol{\pi}/\mathbf{z})). \tag{11}$$

Hence, we may write $\boldsymbol{\pi}(\mathbf{z})$ in light of (11) indicating that $\boldsymbol{\pi}$ only depends on $\mathbf{z}$.

Finally, we show that a benefit-by-cost type social welfare metric can be assessed by a bundle of local innovations, measured by a bundle of productivity gains $\mathbf{z}$. Suppose that the ex-post social welfare ($\mathcal{W}$) is a bundle of commodity-wise effective outputs per cost, i.e.,

$$\mathcal{W} = \mathcal{W}\left(\frac{\lambda_0 y_0}{\rho_0\pi_0 y_0}, \frac{\lambda_1 y_1}{\rho_1\pi_1 y_1}, \cdots, \frac{\lambda_n y_n}{\rho_n\pi_n y_n}\right) = \mathcal{W}\left(\mathbf{z}/\boldsymbol{\pi}\right), \tag{12}$$

where y_0 denotes primary factor inputs. From (11) and (12), we see that $\mathcal{W}$ depends only on $\mathbf{z}$. Moreover, we define welfare gain as the exponential of the welfare growth,

$$e^{\Delta \ln \mathcal{W}} = \mathcal{W}^1/\mathcal{W}^0 = \mathcal{W}(\mathbf{z}/\boldsymbol{\pi})/\mathcal{W}(\mathbf{1}/\mathbf{p}). \tag{13}$$

Hence, welfare gain is the commodity-wise ratio of the benchmark and ex-post welfares. Note that $\mathbf{p}$ is the benchmark equilibrium price under $\mathbf{z} = \mathbf{1}$, such that $\mathbf{p} = \mathbf{h}(\mathbf{p})$, which is available from the benchmark state onwards.

3.2 Structural propagation

Technological structure refers to the physical input–output structure given by an $n \times n$ matrix $\boldsymbol{\Xi} = (\boldsymbol{\xi}_1, \cdots, \boldsymbol{\xi}_n)'$, where $\boldsymbol{\xi}_i = (x_{i1}/y_1, \cdots, x_{in}/y_n)$, along with the primary input coefficients vector $\boldsymbol{\xi}_0 = (x_{01}/y_1, \cdots, x_{0n}/y_n)$. The technological structure, ex-post of

[10] The underlying price $\boldsymbol{\pi}$ is the equilibrium price when all commodities are standardized ($\boldsymbol{\rho} = 1$) and quantified in efficiency units.

exogenous local innovation (given as productivity gain) $\mathbf{z}$ is Ξ, under the equilibrium solution of (11). The ex-post π can be obtained recursively via (11) for a given $\mathbf{z}$, because any unit cost function is strictly concave with respect to its argument (in this case, π). Solving for π, we normalize the standard wage ($\rho_0\pi_0 = 1$) according to the benchmark (p_0), so that $\rho_0 = \pi_0 = p_0 = 1$. We let π and ρ be n dimensional vectors. Let us also assume that $z_0 = 1$, for convenience, so that we can redefine $\mathbf{z}$ as an n dimensional vector.

The technological structure can be obtained via Shephard's lemma. That is,

$$\Xi(\mathbf{z}) = \begin{bmatrix} \frac{\partial \rho_1 h_1(\pi/\mathbf{z})}{\partial \rho_1 \pi_1} & \cdots & \frac{\partial \rho_n h_n(\pi/\mathbf{z})}{\partial \rho_1 \pi_1} \\ \vdots & \ddots & \vdots \\ \frac{\partial \rho_1 h_1(\pi/\mathbf{z})}{\partial \rho_n \pi_n} & \cdots & \frac{\partial \rho_n h_n(\pi/\mathbf{z})}{\partial \rho_n \pi_n} \end{bmatrix} = \langle \rho \rangle^{-1} \nabla \mathbf{h}(\pi/\mathbf{z}) \langle \rho \rangle$$
$$= \langle \pi \cdot \rho \rangle^{-1} \mathbf{A} \langle \pi \cdot \rho \rangle, \tag{14}$$

where the angled brackets represent diagonalization. $\mathbf{A}$ denotes the ex-post cost share matrix. Note that the coefficients of the primary input are $\xi_0 = \nabla \mathbf{h}_0(\pi/\mathbf{z}) \langle \rho \rangle = \mathbf{a}_0 \langle \pi \cdot \rho \rangle$, because we normalize the standard wages of different periods to unity. Equation (14) shows that a bundle of local innovations $\mathbf{z}$ propagates through the economy recursively with respect to (11), until it finds an equilibrium technological structure.

Consider the case when social welfare is assessed by benefits and costs, where benefits constitute the benchmark final demand denoted $\mathbf{d} = (d_1, \cdots, d_n)$, and costs are the sum of ex-post sector-wise primary inputs denoted by $L = L_1 + \cdots + L_n$. Note that L depends upon $\mathbf{d}$ and $\mathbf{z}$.

Under the input–output framework, the ex-post relationship between $\mathbf{d}$ and L can be written as

$$L = \xi_0 [\mathbf{I} - \Xi]^{-1} \langle \lambda \rangle^{-1} \mathbf{d}' = \mathbf{a}_0 \langle \pi \cdot \rho \rangle \left[\mathbf{I} - \langle \pi \cdot \rho \rangle^{-1} \mathbf{A} \langle \pi \cdot \rho \rangle \right]^{-1} \langle \lambda \rangle^{-1} \mathbf{d}',$$

given that the ex-post final demand equivalent of the fixed benchmark final demand (d_j) is d_j/λ_j for all j. After some manipulation regarding (9), this is reduced to

$$L(z; \mathbf{d}) = \mathbf{a}_0 [\mathbf{I} - \mathbf{A}]^{-1} \langle \pi/\mathbf{z} \rangle \mathbf{d}' = (\pi/\mathbf{z}) \mathbf{d}'. \tag{15}$$

We can use (15) to measure the benefit-by-cost social welfare. Here, we use the reciprocal of primary inputs (costs) required to produce certain amount of outputs (benefits) as the measure of social welfare. That is,

$$\mathcal{W}(\mathbf{z}/\pi) = 1/L(z; \mathbf{d}) \tag{16}$$

Given that the benchmark sum of primary inputs is $L(\mathbf{1}; \mathbf{d}) = \mathbf{pd}'$, according to (15), the welfare gain can be assessed as follows:

$$\frac{\mathcal{W}^1}{\mathcal{W}^0} = \frac{\mathcal{W}(\mathbf{z}/\pi)}{\mathcal{W}(\mathbf{1}/\mathbf{p})} = \frac{\mathbf{pd}'}{(\pi/\mathbf{z})\mathbf{d}'} \tag{17}$$

Furthermore, in case we want to study the sector-wise distribution of the propagated primary inputs for (15), we use $\mathbf{L} = (L_1, \cdots, L_n)$ where $\mathbf{L1}' = L$, as below:

$$\mathbf{L}(z; \mathbf{d}) = \mathbf{a}_0 \left\langle [\mathbf{I} - \mathbf{A}]^{-1} \langle \pi/\mathbf{z} \rangle \mathbf{d}' \right\rangle. \tag{18}$$

3.3 Propagation under different functional forms

To further investigate the structural propagation in (16), let us be more specific about the types of production functions. The profit maximizing problem of a producer in the jth industry under a Cobb–Douglas technology is

$$\max_{y_j, x_{ij}} \rho_j \pi_j y_j - \sum_{i=0}^{n} \rho_i \pi_i x_{ij} \quad \text{s. t.} \quad \lambda_j y_j = z_j \prod_{i=0}^{n} \left(\lambda_i x_{ij}\right)^{a_{ij}}, \tag{19}$$

where a_{ij} denotes j's output elasticity for the ith input, with a constant-returns-to-scale technology, such that $\sum_{j=0}^{n} a_{ij} = 1$.[11] The above problem is compatible with the unit cost functions for both the ex-post and benchmark, that is,

$$\rho_j \pi_j = \rho_j \prod_{i=0}^{n} \left(\frac{\pi_i}{z_i a_{ij}} \right)^{a_{ij}}, \quad p_j = \prod_{i=0}^{n} \left(\frac{p_i}{a_{ij}} \right)^{a_{ij}}. \tag{20}$$

Note that the benchmark price $\mathbf{p} = (p_1, \cdots, p_n)$ is the equilibrium price $\boldsymbol{\rho\pi} = (\rho_1 \pi_1, \cdots, \rho_n \pi_n)$ at the benchmark state, i.e., $\mathbf{z} = (1, \cdots, 1)$.

To solve (20) for $\boldsymbol{\pi}/\mathbf{z}$, we take the log and subtract the equations to obtain

$$\ln \pi_j - \ln p_j = \sum_{i=0}^{n} a_{ij} \left(\ln \pi_i - \ln p_i - \ln z_i \right). \tag{21}$$

Rewriting (21) for an $n \times n$ multiple-industry setting using matrices, we get

$$\ln \boldsymbol{\pi} - \ln \mathbf{p} = \left[\ln \boldsymbol{\pi} - \ln \mathbf{p} - \ln \mathbf{z} \right] \mathbf{A},$$

where we abbreviate, for example, $\ln \boldsymbol{\pi} = (\ln \pi_1, \ldots, \ln \pi_n)$. After some manipulation, we obtain

$$\boldsymbol{\pi}/\mathbf{z} = \mathbf{p} \cdot \exp\left(- (\ln \mathbf{z}) \left[\mathbf{I} - \mathbf{A} \right]^{-1} \right). \tag{22}$$

Hence, combining (22) with (18) produces a formula for assessing the distribution of primary inputs, ex-post of exogenous local innovations ($\mathbf{z}$) under the Cobb–Douglas technology. That is,

$$\mathbf{L}(\mathbf{z}; \mathbf{d}) = \mathbf{a}_0 \left\langle [\mathbf{I} - \mathbf{A}]^{-1} \left\langle \mathbf{p} \cdot \exp\left(- (\ln \mathbf{z}) [\mathbf{I} - \mathbf{A}]^{-1} \right) \right\rangle \mathbf{d}' \right\rangle. \tag{23}$$

Accordingly, in regard to (17) and (22), we have the welfare gain for a Cobb–Douglas technology:

$$\frac{\mathcal{W}^1}{\mathcal{W}^0} = \frac{\mathbf{p} \left\langle \exp\left((\ln \mathbf{z}) [\mathbf{I} - \mathbf{A}]^{-1} \right) \right\rangle \mathbf{d}'}{\mathbf{p}\mathbf{d}'}. \tag{24}$$

Next, we examine a Leontief technology. In this case, there is no technology substitution and, therefore, no structural propagation. We start with the ex-post equilibrium monetary balances for an industrial sector as described below:

$$\rho \pi \frac{y}{\lambda} = \frac{1}{z} \left(\rho_0 \pi_0 \frac{x_0}{\lambda_0} + \rho_1 \pi_1 \frac{x_1}{\lambda_1} + \cdots + \rho_n \pi_n \frac{x_n}{\lambda_n} \right).$$

By using the benchmark physical input-output coefficients $\xi_{ij} = x_{ij}/y_j$ that are invariant particularly in this case, and in regard to (9), this reduces to the following equation:

$$\boldsymbol{\pi} = \boldsymbol{\xi}_0 + \boldsymbol{\pi} \langle \mathbf{z} \rangle^{-1} \boldsymbol{\Xi}$$

Now, since $\boldsymbol{\Xi}(\mathbf{1}) = \langle \mathbf{p} \rangle^{-1} \mathbf{A} \langle \mathbf{p} \rangle$ according to (14), we obtain:

$$\boldsymbol{\pi}/\mathbf{z} = \boldsymbol{\pi} \langle \mathbf{z} \rangle^{-1} = \boldsymbol{\xi}_0 \left[\mathbf{I} - \langle \mathbf{z} \rangle^{-1} \boldsymbol{\Xi} \right]^{-1} \langle \mathbf{z} \rangle^{-1} = \mathbf{a}_0 \left[\langle \mathbf{z} \rangle - \mathbf{A} \right]^{-1} \langle \mathbf{p} \rangle \tag{25}$$

[11] The first order condition for (19) indicates that a_{ij} agrees with the cost share of the ith input in the jth industry and also with the monetary-based input–output coefficient. Although this coefficient remains fixed, changes to the relative factor price change the physical input–output coefficient, ξ_{ij} (the technological structure), such that $a_{ij} = \left(p_i/p_j\right) \xi_{ij}$, where p indicates price.

Hence, combining (25) with (18) produces a formula for assessing the distribution of primary inputs, ex-post of exogenous local innovations ($\mathbf{z}$) under the Leontief technology. That is,

$$\mathbf{L}(\mathbf{z};\mathbf{d}) = \mathbf{a}_0 \left\langle [\langle \mathbf{z} \rangle - \mathbf{A}]^{-1} \langle \mathbf{p} \rangle \, \mathbf{d}' \right\rangle. \tag{26}$$

Further, in regard to (17) and (26), we have the welfare gain for the Leontief technology:

$$\frac{\mathcal{W}^1}{\mathcal{W}^0} = \frac{\mathbf{p}\mathbf{d}'}{\mathbf{a}_0 \, [\langle \mathbf{z} \rangle - \mathbf{A}]^{-1} \langle \mathbf{p} \rangle \, \mathbf{d}'}. \tag{27}$$

Contrary to these two cases, structural propagation for a constant elasticity of substitution (CES) technology cannot be reduced to a closed form. Thus, we used a numerical approach. Table 2 shows the primary input change i.e., $\Delta \mathbf{L}(\mathbf{z}_{\mathrm{PO}}) = \mathbf{L}(\mathbf{1}) - \mathbf{L}(\mathbf{z}_{\mathrm{PO}})$ triggered by a doubling of the port operation productivity ($\mathbf{z}_{\mathrm{PO}} = (1, \cdots, z_{\mathrm{PO}}, 1, \cdots, 1)$, where $z_{\mathrm{PO}} = 2$), via different functional forms. The results include CES technology, where we used the parameters estimated by the method presented in Appendix B for non-quality-adjusted deflators. Note that all three models were calibrated using the 395-sector input–output table for Japan, 2005. The first row shows the sum of the differences between the benchmark and ex-post sector-wise primary inputs. Note that the port operation output was 1,452,517 million JPY in 2005 MIAC (2009). These results suggest that Leontief technologies produced the smallest welfare gain estimates, reflecting an inflexible production technology. The second row shows the kurtosis of the sectoral distribution of the changes in the primary inputs. Notably, the sector-wise distribution is polarized (with regard to the relative magnitude of the kurtosis, κ) for Leontief, than the other two functional forms, where the technologies are assumed to be flexible.

3.4 Example: R&D investment allocation

Given that quality-adjusted productivity gain is available for all industries, we can consider an industry-wise allocation of R&D investment that encourages local innovations and maximizes the social welfare gain. Suppose that the local innovation (i.e., quality-adjusted productivity gain, z) and local R&D investment (r) satisfy

$$\ln z = kr, \tag{28}$$

where k denotes a parameter that can be measured by observing the actual values of r and z. Note that no local R&D investment ($r = 0$) implies no local innovation ($z = 1$).

Then, we can use the R&D investment allocation problem to find an allocation that maximizes the potential social welfare gain or that minimizes the labor required to produce and consume some given final demand $\mathbf{d}$, subject to budget constraints. With regard to (23), the problem, under Cobb–Douglas technologies, is

Table 2 Primary input savings by port operation productivity doubling in different functional forms. (unit: Million JPY)

	Cobb–Douglas	Leontief	CES
$\Delta \mathbf{L}\mathbf{1}'$	927,494	726,101	875,729
Kurtosis	(114)	(342)	(182)

$$\min_{\mathbf{r}} L = \mathbf{a}_0 \left[\mathbf{I} - \mathbf{A}\right]^{-1} \left\langle \mathbf{p} \cdot \exp\left(-\left(\mathbf{k} \cdot \mathbf{r}\right)\left[\mathbf{I} - \mathbf{A}\right]^{-1}\right)\right\rangle \mathbf{d}' \quad \text{s. t.} \quad B \geq \mathbf{r}\mathbf{1}', \tag{29}$$

where B denotes the budget constraint for R&D investments. The allocation of R&D investment is $\mathbf{r} = (r_1, r_2, \ldots, r_n)$.

Before presenting an example, let us examine the solvability of the problem in (29). Instead of presenting a full proof that shows the convexity of L with respect to $\mathbf{r}$, we consider a two-industry version of the problem:

$$\min_{r_1, r_2} L = b_1 e^{-c_{11} r_1 - c_{12} r_2} + b_2 e^{-c_{21} r_1 - c_{22} r_2} \quad \text{s. t.} \quad B \geq r_1 + r_2. \tag{30}$$

Note that $b_i > 0$ and $c_{ij} > 0$ summarize the non-variables that cover everything except $\mathbf{r}$ in the objective function of (29). We can examine the convexity of L with respect to r_1 and r_2, by determining if the Hessian of L is positive semi-definite. The principal minors of this Hessian are

$$\frac{b_1 c_{11}^2}{e^{c_{11} r_1 + c_{12} r_2}} + \frac{b_2 c_{21}^2}{e^{c_{21} r_1 + c_{22} r_2}} > 0, \qquad \frac{b_1 b_2 \left(c_{12} c_{21} - c_{11} c_{22}\right)^2}{e^{(c_{11} + c_{21}) r_1 + (c_{12} + c_{22}) r_2}} \geq 0.$$

These signs indicate that L is indeed convex with respect to r_1 and r_2, which means that problem (30) has a unique solution (although the solution may be a corner).

Now, we present a three-industry example with data taken from the 2005 input–output table for Japan MIAC (2009). The input–output coefficient matrix $\mathbf{A}$, the primary factor coefficient $\mathbf{a}_0$, and the final demand vector $\mathbf{p} \cdot \mathbf{d}$ (in trillion JPY) are

$$\mathbf{A} = \begin{bmatrix} 0.12 & 0.06 & 0.01 \\ 0.19 & 0.42 & 0.09 \\ 0.17 & 0.18 & 0.26 \end{bmatrix}, \quad \mathbf{a}_0 \Big[= \; 0.48 \;\; 0.66 \;\; 0.36 \Big], \quad (\mathbf{p} \cdot \mathbf{d})' = \begin{bmatrix} -13 \\ 154 \\ 358 \end{bmatrix}.$$

The total R&D expenditure was 11 trillion JPY ($B = 11$) in 2005 and was allocated over the three sectors as follows.[12]

$$B = 11, \quad \mathbf{r} = \begin{bmatrix} 0 & 10 & 1 \end{bmatrix}.$$

We must use an ad hoc parameter ($\mathbf{k}$), because the parameters have not yet been estimated. However, using

$$\mathbf{k} = \begin{bmatrix} 0.01 & 0.20 & 0.05 \end{bmatrix}, \quad \mathbf{r} = \begin{bmatrix} 0 & 10 & 1 \end{bmatrix}.$$

we get the observed $\mathbf{r}$ as the solution of the optimization. Note that we can easily solve the nonlinear optimization problem in (29) using affordable computation equipment, if the number of sectors are limited (as in this example).

Next, suppose that the R&D investment of the tertiary industry becomes relatively efficient. Minimizing L, we obtain the new solution $\mathbf{r}^{\dagger}$. Hence, it is appropriate that some of the R&D resources in the secondary industry are allocated to the tertiary industry.

$$\mathbf{k}^{\dagger} = \begin{bmatrix} 0.01 & 0.20 & 0.06 \end{bmatrix}, \quad \mathbf{r}^{\dagger} = \begin{bmatrix} 0 & 8 & 3 \end{bmatrix}.$$

Alternatively, if the R&D investment of the tertiary industry becomes relatively less efficient, we obtain the corner solution $\mathbf{r}^{\ddagger}$, such that the concentration of R&D resources on the secondary industry is optimum.

$$\mathbf{k}^{\ddagger} = \begin{bmatrix} 0.01 & 0.20 & 0.04 \end{bmatrix}, \quad \mathbf{r}^{\ddagger} = \begin{bmatrix} 0 & 11 & 0 \end{bmatrix}.$$

[12] We ignore (assume null) the common R&D investment for labor in industries such as education and training, because it is difficult to divide aggregated R&D investments into sector-specific and common parts.

4 Conclusions

In this article, we developed a relevant link between R&D investment and its final outcome, social welfare gain. The first stage of this link corresponds to R&D and innovation that involves qualitative and quantitative improvements in the production of commodities. Because we considered industry-wise R&D investment, we focused on the industry-wise local innovation that contributes to each industry, while eliminating any foreign contribution from the input factor. We found that local innovation can be measured by the quality-adjusted productivity gain, which considers quantitative and qualitative improvements in the inputs and the outputs of production, under the efficiency unit approach. Moreover, we showed how quality-adjusted productivity gain can be measured using two cost share accounts (input–output tables) of an economy and a quality-adjusted deflator.

The second stage of the link corresponds to local innovation and its social welfare gain. In an opposite manner to the first stage, we considered the entire feedback by not avoiding the technological interdependencies among industries, initiated by the exogenous productivity gain that reflects local innovation. We call this feedback structural propagation, because the productivity gain alters the prices of the outputs of industries who can alter their technologies in response to price changes. When modeling the structural propagation, we naturally considered both qualitative and quantitative aspects of productivity. The ex-post structure allows us to assess the social welfare gain initiated by the introduction of local innovation.

Although we have demonstrated how the welfare maximizing allocation of R&D investment could be obtained under a Cobb–Douglas technology (where the structural propagation can be considered using a closed form), we used ad hoc parameters because we did not have quality-adjusted productivity measurements. Clearly, an important future task will be to measure industry-wise quality-adjusted deflators to obtain relevant productivities. We could then estimate reliable CES parameters for more factual (and less restricted) technologies, and perhaps further econometric assessments of innovation, using the structural propagation analyses outlined in this paper.

Appendix A: Measurement of quality-adjusted deflator

Given that input–output accounts (Y and X) and the cost shares α (input–output coefficients) are available every 5 years, the measure of quality-adjusted productivity gain or growth ultimately depends on the measure of $\boldsymbol{\mu}$, which is the quality-adjusted deflator for all industrial categories (sectors), according to (7). As far as the durable final commodities are concerned, we can measure $\boldsymbol{\mu}$ using the hedonic approach Rosen (1974), where we regress the price of the commodity on its attributes. A typical hedonic regression formula (with zero intercept) is

$$p_i = \boldsymbol{\beta}\mathbf{q}_i + \varepsilon_i = \beta_1 q_{1i} + \beta_2 q_{2i} + \cdots + \varepsilon_i,$$

where p_i is the price of the ith commodity, q_{li} is the lth Lancasterian attribute of the ith commodity, β_l is the hedonic marginal price of the lth attribute, and ε_i is the disturbance term. Let $\mathbf{b}^0 = \left(b_1^0, b_2^0, \cdots\right)$ and $\mathbf{b}^1 = \left(b_1^1, b_2^1, \cdots\right)$ denote the estimated hedonic marginal prices for the benchmark and the ex-post, respectively. Now, for the same (standard) set of attributes $\bar{\mathbf{q}} = (\bar{q}_1, \bar{q}_2, \cdots)'$, we can obtain the benchmark and ex-post price estimators for a standard commodity (with a set of standard attributes), i.e., $\hat{p}^0 = \mathbf{b}^0\bar{\mathbf{q}}$ and $\hat{p}^1 = \mathbf{b}^1\bar{\mathbf{q}}$. A quality-adjusted deflator can then be evaluated by $\bar{\mu} = \hat{p}^1/\hat{p}^0$.[13]

However, the hedonic approach is not applicable when Lancasterian attributes are not observed in the commodities. Typically, we cannot observe Lancasterian attributes in service commodities. We may also not be able to observe attributes for the intermediate commodities, simply because the data is too expensive. In these cases, we can instead use the market share to study the quality-standardized deflator. Under the discrete choice theory McFadden (1973), a market share reflects the representative consumer's level of utility drawn from the alternatives. In particular, a logit formula connects the market share s_i of an alternative i with the mean utility V_i (with a Gumbel-distributed disturbance) from consuming i, in such a way that $s_i = e^{V_i}/\sum_j e^{V_j}$. Following Berry (1994), we write this formula as

$$\ln s_i - \ln s_0 = V_i,$$

where we normalize the mean utility of the outside good to zero ($V_0 = 0$)

Further, we let the mean utility be subject to a linear function of the attributes of the commodity, i.e., $V_i = \gamma p_i + \beta q_i$, where q_i denotes the quality that is unobservable to the econometrician. Note that γ is the marginal disutility of payment, which can be interpreted as the marginal utility of the income of a representative consumer. We may then assess the quality-adjusted deflator (μ_i) using market shares for the benchmark and the ex-post via

$$\mu_i = \frac{p_i^1/q_i^1}{p_i^0/q_i^0} = \frac{\left(\ln s_i^0 - \ln s_0^0 - \gamma p_i^0\right) p_i^1}{\left(\ln s_i^1 - \ln s_0^1 - \gamma p_i^1\right) p_i^0}.$$

We may set the marginal utility of the income of a representative consumer to unity ($\gamma = 1$), as in Bresnahan (1987), or use the estimated values, as in Layard et al. (2008).

Appendix B: Multi-factor CES production functions

A multi-factor CES production function of an industry (index j omitted) takes the form

$$y = f(\mathbf{x}; z, \boldsymbol{\delta}, \sigma) = z\left(\sum_{i=0}^{n} \delta_i^{\frac{1}{\sigma}} x_i^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

where we want to estimate the share parameter ($\delta_i > 0$) for the ith input and the elasticity of substitution $\sigma \geq 0$, which is unique across the input factors. The cost shares for the ith input under CES, which can be monitored for different periods, are

$$a_i^0 = \delta_i \left(z^0 p^0/p_i^0\right)^{\sigma-1}, \quad a_i^1 = \delta_i \left(z^1 p^1/p_i^1\right)^{\sigma-1}. \tag{31}$$

Note that the parameters δ_i and σ are assumed to be constant over time, but there is only a small chance that these identities are simultaneously true.

We may try to find the best fitting parameters, i.e., δ_i and σ, as in Nishimura (). We first rewrite (31) to describe the share parameter δ_i as a function of σ that is consistent with the observations for two periods. That is,

$$\delta_i(\sigma; 0) \equiv \alpha_i^0 \left(z^0 p^0/p_i^0\right)^{1-\sigma}, \quad \delta_i(\sigma; 1) \equiv \alpha_i^1 \left(z^1 p^1/p_i^1\right)^{1-\sigma}.$$

[13] The quality-adjusted deflator $\bar{\mu}$ depends on the standard commodity, and we may use the average to calculate the quality-adjusted productivity gains.

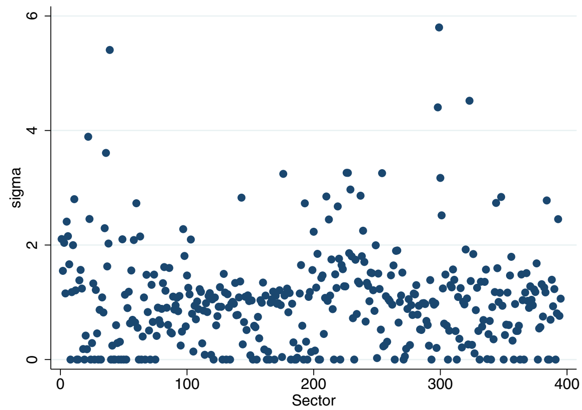

Fig. 2 Estimates of CES marginal elasticity of substitution for various industrial sectors (σ_j) based on the official deflator, for 2000–2005, in Japan

These parameters are constant *per se*, so we search for the σ that means these two parameters are as close as possible. That is,

$$\sigma = \arg\max_{\sigma \geq 0} S\left(\boldsymbol{\delta}\left(\sigma; 0\right) - \boldsymbol{\delta}\left(\sigma; 1\right)\right)$$

where $S\left(\mathbf{u}, \mathbf{v}\right)$ is some similarity function (such as cosine, correlation, etc.) between two vectors $\mathbf{u}$ and $\mathbf{v}$. After estimating the elasticity of substitution σ, we can choose the share parameters $\boldsymbol{\delta}$ to fit the observed metrics. Figure 2 shows the estimated values for 395 industrial sectors using the Japanese input–output tables for 2000 and 2005 MIAC (2009), with the objective function being the correlation. Note that the productivity gains used in these calculations are not fully quality-adjusted, but quantity-adjusted. In Fig. 2, the production functions were estimated to be either Leontief ($\sigma = 0$), Cobb–Douglas ($\sigma = 1$), or otherwise.

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